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An Evaluation of the Dividend Discount Model and its Extensions

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Abstract: The valuation of the stock is one of the most fundamental financial concepts, whereas the Dividend Discount Model is the basis of the stock valuation. The model state that the value of the stock is the sum of the present value of the infinite future dividend. It is, however, impossible for Humans to forecast future dividend accurately. Therefore, the assumption of the future dividend and the extension of the dividend discount model are required. In this paper, the assumption, advantages, and limitations of the general Dividend discount model and its extension, including the Fix dividend discount model, Gordon growth model, Two-stage model, Three-phase model, H-model, Geometric and additive model, and the modified Geometric and additive model, are introduced in this paper, which can be used as a reference for the investors for them to make their financial decision or for the scholars for their researches.

Keywords: Dividend Discount Model, Advantages, Limitations, review, Gordon growth model.

1. Introduction

1.1. Background

Stock valuation is one of the most basic financial concepts. The price of a stock of a company which can be seen in the stock market, is generally the extrinsic value of that stock, which is determined by supply and demand. The question of how to study the intrinsic value of a stock has been under discussion. The Dividend Discount Model (DDM) is one of the great ways to discuss and estimate the intrinsic value of a stock, so some scholars have proposed using the dividend discount model to solve this problem. It has been recognized as one of the basic models for valuing the intrinsic value of a stock. In the 1930s, Williams and Gordon proposed this model [1]. Its extension and emergence created the theoretical basis for the quantitative analysis of fictitious capital, assets and company value. Additionally, it provided a reasonable basis for the analysis of equity investment. The theoretical basis of DDM is the present value principle.

The "real" or "intrinsic" value of any asset, in general, is the cash flow that the investor who owns the asset will receive over a period of time [2]. In terms of the estimation of the stock, the stockholders are expected to receive dividends and capital gain. However, to determine the value of the stock, the

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dividends and the capital gain cannot be simply added up since the stream of dividend and the capital gain is not expected to receive at the same time. To understand the statement better, a little example might be helpful: Assuming that there are two people, A and B. A lends $100 to B for 1 year. A goes to B for money after one year. B gives me two choices: I'll give you $100 now, or I'll give you $100 a year from now. A must have chosen the first option because if A gets the money, A could deposit it in his bank account, which can provide an interest rate of 2% annually. After one year, A will have $102. Therefore, the intrinsic value of the stock should be equal to the present value of the dividends and capital gain, instead of the Future value. Since the capital gain is determined by the future dividends, the intrinsic value can equal the sum of the present value of its infinite future dividends.

1.2. Limitations

Due to the fact that the DDM is a wireless dividend stream model, year-by-year dividend projections are required. Therefore, forecasting into the distant future is a must, which is inaccurate, open-ended and inconvenient in reality. It is impossible for a human to predict the distant future. One may be able to anticipate future dividends for a short period, but one cannot anticipate them indefinitely, which leads to the limitations of DDM. Due to the limitation that one cannot predict future dividends with certainty, it is necessary to propose some reasonable assumptions to reduce the complexity of DDM so that the value of the stock can be evaluated by using the DDM.

1.3. Objective

Although countless scholar has proposed their assumption and ended with their models as the extensions of DDM, it is still a lack of an overview of those extensions. Therefore, the aim of this paper is to introduce DDM and its extensions along with their limitation and advantages, which can be used as a reference for investors or researchers.

2. General dividend discount model

The valuation of the stock is one of the most basic topics of finance. The value of the stock is not the price that can be seen in the stock market as it is usually determined by the demand and the supply. Therefore, it is common that there is sometimes an under or overpriced stock in the market. Identifying such stocks becomes a vital problem for investors. Willian proposed a way to determine the intrinsic value of the stock in 1938[1]. He states that the intrinsic value of the stock should be determined by its future cash flows, which are the dividend payments and the future price when selling the stock. Instead of simply adding those two cash flows, the value of the stock should be equal to the sum of the present value of the dividend and the future price, as the future cash flows, in general, are not expected to receive while purchasing it.

Assuming that one purchases a stock while period t and going to sell it at period t + 1. The intrinsic value of the stock can be expressed mathematically as:

$$P_t^D = \frac{D_{t+1}+P_{t+1}}{1+r_{t+1}} \tag{1}$$

where P_t^D is the intrinsic value of the stock estimated by the general dividend discount model; D_{t+1} is the dividends paid in period t; P_{t+1} is the price of the stock in period t + 1; the r_{t+1} is the discount rate in period t + 1. Assuming that the stock will be held indefinitely, the future price of the stock will approach 0 when the time period tends to the infinite, which can be expressed mathematically as:

$$\lim_{n \to \infty} P_{t+n}^D = \lim_{n \to \infty} \frac{P_{t+n+1}}{\prod_{j=1}^{n+1}(1+r_{t+j})} = 0 \tag{2}$$

Therefore, the intrinsic value of the stock is equal to the present value of its infinite future dividends, which can be expressed mathematically as:

$$P_t^{\mathrm{D}} = \sum_{i=1}^{+\infty} \frac{D_{t+i}}{\prod_{j=1}^{i}(1+r_{t+j})} \quad (3)$$

2.1. Advantages

The Dividend discount model provides a possible way to estimate the intrinsic value of the stock by calculating the net present value of all of its future dividends. It could help the investors to identify the under or overestimate stock so that they can make wise financial decisions. Nevertheless, the limitation of this model is also obvious.

2.2. Limitation

It can be easily seen from equation (3) that the value of the stock is determined by its future dividend and the discount rate. This means that to estimate the value of the stock, it needs the exact value of every year's dividends or discount rate, leading the entire model to become extremely complex and nearly impossible to calculate. To overcome this limitation, many scholars proposed various assumptions about the patterns of the expected dividends and end up with extensions of the general dividend discount model. However, most of the extensions all assume that the discount rate is a constant, which is unlikely to happen when estimating the value of a long-run stock.

3. Extensions

3.1. Gordon growth model

The Gordon growth model (GGM) proposed a famous assumption for reducing the complexity of the DDM. GGM was developed by Shapiro (1956) and Gordon (1962) and was first published by Gordon in 1959 [3]. In order to reduce the complexity of the general DDM, the model assumed that the expected dividends of the company would grow at a constant rate, which may be expressed mathematically as:

$$D_{t+1} = D_t(1+g) \quad (4)$$

where g represents the constant growth rate of the dividends, Dt is the dividends paid in period t. Under this assumption and assuming the discount rate is a constant, Equation 1 can be rewritten as:

$$\mathrm{P}_{\mathrm{t}}^{G} = \frac{D_{\mathrm{t}+1}}{r-g} = D_{\mathrm{t}}\frac{1+g}{r-g} \quad (5)$$

where $\mathrm{P}_{\mathrm{t}}^{\mathrm{G}}$ is the intrinsic value of the stock estimated by the GGM, Dt + 1 is the expected dividends in period t + 1; r is the discount rate; g is the growth rate in dividends in the infinite future. Notably, r and g are both constant [4].

Obviously, it is essential to have a reasonable estimate of the growth rate when using GGM to estimate the intrinsic value of the stocks of a company. In general, the product of return on equity and the profit retention ratio is used to estimate the growth rate, which can be expressed mathematically as:

$$g = b \times ROE \quad (6)$$

where b is the profit retention ratio; ROE is the return on equity which can be determined by multiplying the asset-to-equity ratio times the return generated on assets [5].

Advantages. Due to the fact that GGM reduces the complexity of the general DDM, it helps countless investors and researchers find the intrinsic value of the stock to assist them in making financial decisions or helping them with their research. As a hypothetical example, there is a company named XYZ whose stock price is $ 90 per share in period 0. The company paid a $5 dividend per share this year, which is expected to grow 6% annually. The current discount rate is 10%. The calculation of the intrinsic value of the stock is shown as follows:

$$\mathrm{P}_0^G = \$5 \times \frac{1+.05}{.10-.05} = \$105 \tag{7}$$

Therefore, according to the GGM, the intrinsic value of the stock is higher than the current stock price, which means that it would be a good idea to purchase the stock of XYZ now.

Limitations. However, there are lots of limitations when using this model. First of all, it is criticized for its assumptions, especially the assumption that growth is both geometrical and indefinite. This model would be beneficial for some mature and stable companies [6]. However, the dividends are unlikely to remain the same for most companies. The unexpected financial difficulties or success may lead to the fluctuation of the dividends. Moreover, GGM will not function when the discount rate is lower than the growth rate, as the present value of the expected dividends becomes negative, which shows that the model becomes worthless. When the discount rate is equal to the growth rate, the model will not function as well, as the present value of the expected dividends will become infinite when the growth rate approaches the discount rate.

3.2. Two-stage model

The assumption of GGM that the expected dividends will grow at a constant rate is too absolute, so it is hard to happen in real life. In order to make the model applicable to a broader range of companies, Malkiel proposed a new assumption in 1963[7]. The hypothesis assumes that the dividend will grow at a higher rate in the first n years and will start growing at a constant rate. Therefore, by its assumptions and assuming that the discount rate is a constant, the value of the stock is equal to the sum of the present value of the dividend in the first n years and the present value of the stock price in period n, which can be calculated by the GGM, which can be shown mathematically as:

$$P_{\mathrm{t}}^{2\mathrm{S}} = D_0 \sum_{i=1}^{n} \frac{1+g_h}{1+r} + \frac{P_n^{\mathrm{G}}}{(1+r)^n} \tag{8}$$

where $\mathrm{P}_{\mathrm{t}}^{2\mathrm{S}}$ is the estimated value (by two-stage model) of the stock in period t; gh is the growth rate in the first n years, $\mathrm{P}_{\mathrm{n}}^{\mathrm{G}}$ is the estimated value (by GGM) of the stock in period n. Summing the equations and substituting the results of the GGM estimate of $\mathrm{P}_{\mathrm{t}}^{\mathrm{G}}$, the equation will become:

$$P_{\mathrm{t}}^{2\mathrm{S}} = \frac{D_{\mathrm{t+1}}\left[1-\left(\frac{1+g_h}{1+r}\right)^n\right]}{r-g_h} + D_{\mathrm{t}}\left(\frac{1+g_h}{1+r}\right)^n\left(\frac{1+g}{r-g}\right) \tag{9}$$

where gh is the higher constant rate; g is the constant growth rate from period n till forever; Dt+1 is the expected dividend in period 1; Dt is the dividend paid in the most recent 12 months.

Advantages. The Two-stage model is suitable for the company that has a higher growth rate at the early stage. Compared with the GGM, the Two-stage model with two different growth rates can be

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used for a wider range of companies, and its predictions are relatively more accurate for stock values, providing a better reference for investors and researchers.

Limitations. However, there are also some limitations to this model. One of the limitations is that transition period between period n – 1 and n, the growth rate instead of falling from gh to g smoothly, suddenly experiences a drastic drop, which is unlikely to happen in the real life. Additionally, it will be hard to tell when will the higher growth rate drop to the constant growth rate, which means that it will be hard to find a value for n.

3.3. Three-phase model

Since the estimate of two stage model that there will be a sharp drop of the growth rate is unlikely to happened in the real life. To make the model more practical and more realistic, Nicholas Molodovsky proposed a new assumption in 1965 [8]. He assumed that the expected dividend would experience three phases. In the initial phase, the dividend is expected to growth at a higher rate for A years. After Na year the dividend will enter the transition phase (phase 2), where the growth rate of the dividend is expected to decline in a linear fashion for B years. The decline will end at the beginning of phase three, where the dividend is expected to grow at a constant rate forever. Based on this assumption, Fuller discover the three-phase dividend discount model in 1979. The present value of the expected dividends as being written as:

$$P_{\mathrm{t}}^{3p} = \mathrm{PV(Phase\ 1)} + \mathrm{PV(Phase\ 2)} + \mathrm{PV(Phase\ 3)} \tag{10}$$

To be more specific, the equation can also be written as:

$$P_{\mathrm{t}}^{3\mathrm{p}} = D_{\mathrm{t}}\left[\sum_{T=1}^{A}\left[\frac{1+g_a}{1+r}\right]^T + \frac{\sum_{T=1}^{B}\left[\frac{D_{A+T}}{(1+r)^T}\right]}{(1+r)^A} + \frac{D_{A+B}(1+g)}{(r-g)(1+r)^{A+B}}\right] \tag{11}$$

where $\mathrm{P}_{\mathrm{t}}^{3\mathrm{p}}$ is the value of the stock estimated by the three-phase model; ga is the growth rate in the first A year; Da+T is the expected dividend paid in period A + T; DA+B is the expected dividend paid in period A + B;

Advantages. Since the three-phase dividend discount model is capable for the changes in the growth rate in the expected dividend, it is more accurate and practical when it compares to GGM, which assumes that the expected dividend will grow at a constant rate. The growth rate instead of sharply dropping from the higher rate to the constant rate just like the two-stage model does assume that the growth rate will experience a transition period where it is going to decline in a linear fashion to the constant growth rate, which is more realistic and more practical.

Limitations. Due to the complexity of this model, the model is prone to error in the actual use of this model, as it requires the estimates a larger numbers of growth rate when it compares to other extensions. If there are deviations from the true values in the estimation of the growth rate, the payout ratio, the betas, or the length of each phase, the stock values estimated by this model may not be as accurate as the simple two-stage model than it is.

3.4. H-model

To reduce the complexity of the three-phase dividend discount model, Fuller and Hsia proposed the H-model in 1984 [9]. In their assumption, instead of a sudden and dramatic change in the growth rate as the two-stage model assumed, it is expected to decline or increase from a higher or lower growth

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rate in a linear fashion to a "normal" constant growth rate for 2H period. The dividend is then expected to grow at this rate forever. The H-model is illustrated as follows:

$$P_{\mathrm{t}}^{\mathrm{H}} = \frac{D_{\mathrm{t}}(1+g)}{r-g} + \frac{D_{\mathrm{t}}H(g_h-g)}{r-g} \quad (12)$$

where P_t^H is the value of the stock estimate by the H-mode; gh is the higher (or lower) growth rate; H is the half period of the declining (or increasing) process.

Advantages. Compared with the two-stage model, H-model avoids sudden and dramatic changes in the growth rate, which is more practical and more realistic. Although it also greatly reduces the complexity of the three-stage models, the accuracy did not reduce much as it does the complexity.

Limitations. Although it is not much less accurate compared to the three-stage model. But when higher accuracy or estimation of more complex company stocks is required, the H-model may not be the best choice. Additionally, H-model, as well as GGM, two-stage model, and three-phase model, all assume a specific pattern of the growth rate of the expected dividend. However, even though those models are capable for many companies, there will always be some incompatible cases to any of the models mentioned above.

3.5. Geometric model and additional model

Instead of assuming the growth rate will follow a specific pattern, just like the model mentioned above, Hurley and Johnson [10] proposed completely new assumptions and ended up with two new models in 1994. Their model is formulated based on a concept called the "Markov dividend stream". In this concept, the future dividends are expected to have a positive probability of p to grow a certain amount and a probability of 1 – p to remain the same. They also assume that the dividends are expected to grow geometrically or additively. Based on these assumptions, they end up with two equations. Additionally, they also provide a lower bound for each of these equations in the case that there is also some small possibility that the company may go bankrupt.

Additive model. Assuming that the dividend is expected to grow by an amount of C with a positive probability p or stay the same with the probability of 1 – p in period $t \in N*$, which can be shown mathematically as:

$$D_{t+1} = \begin{cases} D_t + C, \ with\ prob\ p \\ D_t, \ with\ prob\ 1 - p \end{cases} \quad (13)$$

Based on their assumption, which is called the additive Markov process assumption, they end up with their additive model which is shown as follows:

$$\mathrm{P}_t^A = \frac{D_{\mathrm{t}}}{r} + \left(\frac{1}{r} + \frac{1}{r^2}\right) Cp \quad (14)$$

where P_t^A is the estimated value of the stock (by the Additive model). Or adding the probability of pb that the company might go bankrupt, which can be shown mathematically as follows:

$$D_{t+1} = \begin{cases} D_t + C, \ with\ prob\ p \\ D_t, \ with\ prob\ p \\ 0, \ with\ prob\ p_b \end{cases} \quad (15)$$

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And the equation is:

$$L_A = \frac{D_{\mathrm{t}}(1-p)}{r+P_b} + \left[\frac{1}{r+P_b} + \frac{1}{(r+p_b)^2}\right] Cp \tag{16}$$

where L_A is the value of the stock estimated by the lower-bound equation of the Additive model. Notably, when Pb is equal to 0, L0 will be the same as V0, i.e., equation (16) will be equal to equation (14).

Geomatical model. Assuming that the dividend is expected to grow at a rate of g with a positive probability p or stay the same with the probability of 1 – p in period $t \in N*$, which can be shown mathematically as:

$$D_{t+1} = \begin{cases} D_t\ (1\ +\ g),\ with\ prob\ p \\ D_t,\ with\ prob\ 1\ -\ p \end{cases} \tag{17}$$

Based on their assumption, which is called the geometrical Markov process assumption, they end up with their additive model which is shown as follows:

$$\mathrm{P}_t^{Go} = \frac{D_t(1+pg)}{r-pg} \tag{18}$$

Or adding the probability of pb that the company might go bankrupt, which can be shown mathematically as follows:

$$D_{t+1} = \begin{cases} D_t\ (1\ +\ g),\ with\ prob\ p \\ D_t,\ with\ prob\ p \\ 0,\ with\ prob\ p_b \end{cases} \tag{19}$$

And the equation is:

$$L_{\mathrm{G}} = D_{\mathrm{t}}\left[\frac{1+pg-P_b}{r-(pg-P_b)}\right] \tag{20}$$

Where L_G is the value of the stock estimated by the lower-bound equation of the Geometric model. Notably, when Pb is equal to 0, Lt will be the same as Pt, i.e., equation (18) will be equal to equation (20).

Advantages. The model proposed by Hurley and Johnson is more flexible than all of the equations cited above, as it allows relatively random changes in the expected dividends, making the model more practical and realistic. Additionally, when this model compares with equation (11), it seems to provide a more accurate estimate but has a lower complexity for the equation.

Limitation. The limitation of this model also cannot be ignored. They assumed that there was a probability that the dividend would grow by a little bit or remain the same. This assumption, however, seems to be incomplete as the dividend may also decrease for some reason, which is not included in their models.

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3.6. Modified geometric and additive model

To overcome the limitation of the geometric and additive model proposed by Hurley and Johnson, Yao proposed a modified model in 1997[11], which is based on a similar assumption as Hurley and Johnson did. Yao added a probability that the dividend is expected to decrease to Hurley and Johnson's assumptions, which be expressed mathematically as:

Modified additive model:

$$D_{t+1} = \begin{cases} D_t + C, \ with\ prob\ p_a \\ D_t - C, \ with\ prob\ p_b \\ \quad Dt, \ with\ prob\ 1 - p_a - p_b \end{cases} \tag{21}$$

Modified geometric model:

$$D_{t+1} = \begin{cases} D_t(1+g), \ with\ prob\ p_a \\ D_t(1-g), \ with\ prob\ p_b \\ \quad Dt, \ with\ prob\ 1 - p_a - p_b \end{cases} \tag{22}$$

Therefore, the equation for those two modified equations are:

Modified additive model:

$$\mathrm{P}_t^{MA} = \frac{D_\mathrm{t}}{r} + \left(\frac{1}{r} + \frac{1}{r^2}\right)(p_a - p_b)C \tag{23}$$

Modified geometric model:

$$\mathrm{P}_t^{MG} = D_\mathrm{t}\frac{1+(p_a-p_b)g}{r-(p_a-p_b)g} \tag{24}$$

where $\mathrm{P}_\mathrm{t}^{\mathrm{MA}}$ and $\mathrm{P}_\mathrm{t}^{\mathrm{MG}}$ is the estimate value of the stock estimated by the modified Additive model and the modified geometrical model, respectively.

Advantages. Since this model includes the probability that the dividend might decrease, the model is more practical and realistic than Hurley and Johnson's.

Limitations. However, this model also has some drawbacks. Firstly, the model did not consider the probability of the company going bankrupt, which might affect the estimation accuracy. Additionally, as well as all of the extensions mentioned above, the models only consider the situation that the dividend is changing geometrically or additively, which might add to the objective factor of the investors or the researchers when using this model. The dividends may also vary in other ways, which these models cannot apply.

4. Conclusions

The Dividend Discount Model had made a significant contribution to the valuation of the stock, which helped countless investors to identify the under or overvalued stocks. Due to the fact that the assumption of DDM that the value of the stock is equal to the sum of the present value of the infinite future dividends is unrealistic as it is impossible to predict every future dividend, the further assumption and the extensional model are required. This paper has introduced the advantages and the limitation of each of the extensions. The Gordon growth model, which assumes that the dividends are

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expected to grow at a constant rate, is the most common valuation model of the stock but is limited to large and mature companies. Two-stage model is able to include two growth rates, but the transition between the higher growth rate and the slower growth rate is sudden and sharp, which is unlikely to happen in this real life. The three-phase model includes a smooth transition phase between the higher and lower growth rates, which can provide a more accurate estimation. Still, the derived equation is too complex, which may cause severe errors in some companies. The H-model has a more straightforward equation and has similar accuracy to the Three-phase model, but when higher accuracy is required, the H-model may not be the best choice. Unlike the model mentioned above, the Geometric and Additive model include the random variable in their model, which assumes that there is a certain probability that the dividend would either increase or remains the same, which is a more realistic assumption. It, however, ignores the probabilities that the dividend might decrease, which the modified geometric and additive model fixes. The limitation of this model, along with all the models mentioned above, is the assumption that the discount rate is a constant, which is unrealistic when estimating a long-time period stock. However, there is still no extension that can perfectly overcome the limitation, a new aspect that future research can work on.

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