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Research and Application on Kelly's Formula

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Abstract: The subject of investment finance holds significant importance in contemporary society, with position management in investment serving as a critical factor in determining outcomes of success or failure. Acquiring knowledge in the realm of efficient asset allocation can significantly enhance individuals' ability to effectively oversee their financial resources. The Kelly formula, initially employed for determining the proportion of funds wagered on a particular game, has emerged as a prominent betting strategy within the realm of probability theory. This method aims to optimize the overall growth rate of the main amount throughout a series of independent bets, each of which is intended to yield positive net returns. This study will examine the historical context of the Kelly formula, its mathematical derivation, its utilization in the realms of gambling and investment, and its consequential implications for individuals' investment strategies.

Keywords: Kelly criterion, derivation, application, investment

1. Introduction

A growth-optimal strategy is characterized by its ability to maximize the anticipated logarithmic growth rate of wealth. In multiple respects, such policies exhibit superior performance compared to alternative investment strategies over an extended period of time. When used in an unending series of investments, a growth-optimal approach exhibits a tendency to amass a significantly larger amount of wealth over time compared to any other investment method. The growth-optimal strategy in the context of a finite sequence of investments aims to maximize the predicted growth rate of wealth while mitigating the risk of bankruptcy. The trajectory of its wealth is characterized by rapid wealth building, accompanied by substantial levels of risk-taking and volatility. The growth-optimal method aims to maximize the logarithmic utility function based on the principles of anticipated utility theory. The successful implementation of a growth-optimal approach in practical settings necessitates the presence of advantageous investment prospects. This entails that the anticipated returns should exhibit positivity, while simultaneously ensuring that the potential losses are constrained within predetermined limits. The origins of contemporary research on growth-optimal strategies may be traced back to Kelly's seminal publication in 1956. In this paper, Kelly introduced a method aimed at maximizing the long-term predicted growth rate of wealth in a multi-period investment context, specifically focusing on compound returns [1].

The Kelly formula is a betting strategy in probability theory that maximizes the long-term growth rate of the principal over independently repeated bets with positive expected net returns. John Larry

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Kelly published the formula in the Bell System Technical Journal in 1956, and it can be used to determine the proportion of money that should be wagered on each game. If the expected net reward of a bet is zero or negative, the Kelly formula advises against betting. However, the Kelly formula has many applications in the investment world, including asset allocation considerations, fund management, accounting for opportunity costs, and non-core assets seeking short-term speculative opportunities. Kelly, the formula's author, was not a seasoned gambler, but a distinguished physicist who invented it while working as a research scientist at the prestigious Bell Laboratories, on the then-emerging frontier of television transmission protocols. The formula is:

$$f = \frac{bp - q}{b} = \frac{p(b+1) - 1}{b}$$

f = fraction of your bankroll you should stake
b = decimal odds
p = probability of winning
q = probability of losing

Kelly's approach borrows from Shannon's work on the scarring of long-distance telephone lines. Kelly shows that Shannon's information theory can be applied here: gamblers do not need complete information. Another of Shannon's colleagues, Edward O. Thorp, applied this formula to blackjack and the stock market. An equivalent idea was proposed by Danny Bernoulli in 1738, but Bernoulli's work was not first translated into English until 1954. But for those who invest only once, choose the portfolio with the highest arithmetic average. This formula is extremely instructive in the gambling and investment fields. The author will discuss and study this formula in a mathematical derivation and in a large number of cases, and explain in detail its application to gambling and investment, respectively.

2. Literature review

Dr. John Kelly of Bell Laboratories was the first to study this problem. He showed that the mathematical model used by Shannon in the theory of communication noise interference is also applicable to the management of risk and return by investors. If the information transfer reduces the error due to noise interference to zero, then, similarly, the investor can reduce the slope length risk to zero while pursuing the maximum compound interest return. While Thorp applied Kelly's formula to the blackjack gambling game, successfully deducing the actual function of Kelly's formula. He applied this formula to various investment opportunities, including blackjack, sports betting, and stock market investments [2]. Another notable study on the Kelly criteria was conducted by Leonard C. MacLean, Edward O. Thorp, and William T. Ziemba. In their 2011 paper "The Kelly Criterion: Theory and Practice", the authors provide a comprehensive overview of the Kelly Criterion and its application to investing. They discuss the assumptions, limitations, and practical considerations of the formula and provide examples of how the formula can be applied to various investment opportunities [3].

In addition, noted investor Warren Buffett has been influenced by Kelly's formula. He simplified and improved it:

$$f = 2p - 1$$

Where: f = percent of investor’s capital to put into a single trade; p = probability of winning.

In fact, the Kelly formula, which Warren Buffett, Charlie Munger and Bill Gross have all cited numerous times, is a formula that calculates the best bet ratio based on the odds of getting the highest

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return on an investment. We could assume a simple bet: you have \$100 to play a coin toss. If the coin is heads, you win \$2 for \$1 (b = 2); if the coin is tails, you lose \$1. What percentage of your capital should you invest in one to maximize your returns?

Historically, the resolution of this inquiry has been perceived as challenging. However, Kelly's formula provides a precise mathematical solution, yielding a value of 25 percent. The study of this formula holds relevance beyond its use in enhancing performance at casinos. It plays a significant role in comprehending the underlying mechanisms through which casinos generate profits. Moreover, it empowers individuals to make more informed decisions regarding asset allocation and fosters the development of a more enlightened financial mindset.

The Kelly criterion has been extensively examined and analyzed in many scholarly investigations and academic publications, affirming its significance and pertinence in the field of investment. The concept of relevance is of significant importance in academic discourse. Despite the formula's inherent limits, its utilization in the management of portfolio risk and enhancement of investment returns renders it a valuable instrument for investors aiming to attain their long-term investment goals.

3. The Kelly criterion

In 1956, Kelly published a groundbreaking paper that applied Shannon's work on information theory to the realm of gambling. The paper laid out strategies for maximizing long-term wealth growth in an infinite number of profitable games. Kelly was inspired to investigate the issue by a general fascination with gambling. At the time of Kelly's article, bookmakers across the United States were placing sports bets on far-flung events, despite being illegal in most states. Kelly's approach was based on the idea that punters receive private and noisy information about the outcome of gambling earlier than bookmakers do, giving them an advantage. In other words, if customers had access to information about the outcome of a sporting event before the bookies did, they could arbitrage profits by betting on players who had already won, to the bookies' detriment. The transmission of information in Kelly time is slow and ambiguous, leading to uncertain outcomes. According to Shannon's information theory, the gambler's private information is transmitted through a noisy channel, which means that it has a probabilistic meaning rather than a definite one.

Kelly's formula, known as the "Kelly Criterion," determines the fraction of wealth that needs to be wagered in order to maximize long-term wealth growth over an endless sequence of favorable investments. This formula can be equally applied to any advantageous investing approach. Kelly's research is grounded in the premise that the private knowledge of a gambler is characterized by uncertainty. However, the gambler possesses the ability to revise their probabilistic views and then make wagers based on these updated beliefs. While private information does not offer a foolproof assurance of the outcome of a sporting event, it does confer an advantage to the recipient compared to ignorant bettors. Gamblers depend on fast and pertinent information regarding matches and their corresponding outcomes, which is acquired through wire services.

The investing approach employed by Kelly aims to optimize the rate of growth of predicted wealth when implemented across a limited series of investments. Kelly's assumption of the noisy channel serves as a prominent illustration of a favorable factor in the realm of gambling. Nevertheless, Kelly's mathematical methodology might be employed in various advantageous investment strategies.

3.1. Derivation of Kelly's formula

Suppose the gambler's capital is N, the betting ratio is f, each game has n outcomes, the net return of the ith outcome is r_i, and the probability of occurrence is p_i. Then the mathematical expectation of the increment (logarithmic growth rate) of the logarithmic capital $\ln N$ after one round is:

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$$\Delta \ln N = \sum_{i=1}^{n} p_i \ln(1 + r_i f) \tag{1}$$

Take the derivative of the above equation with respect to f, and the betting ratio f when taking the extreme value satisfies the equation:

$$\sum_{i=1}^{n} \frac{p_i}{1/r_i + f} = 0 \tag{2}$$

The solution $f = f^*$ that satisfies the above equation (i.e., "Kelly's formula") is the optimal investment ratio. When the expected net return is $\sum_i^n p_i * r_i > 0$, the solution is $f^* > 0$. When the expected rate of return is zero or negative, the bet is usually not allowed. The best strategy is $f = 0$, which means you should not bet to win. If there are only $n = 2$ outcomes per game (win or lose), where $r_1 = r_W > 0$, $r_2 = -r_L < 0$, $p_1 = p$, $p_2 = 1 - p$, then the solution of Kelly's equation is:

$$f^* = \frac{p}{r_L} - \frac{1-p}{r_W} \tag{3}$$

For example, if each game has a winning rate of $p = 40\%$, and the gambler can get the odds 1 to 2 when he wins the game (b = 2), and if he loses the bet, the gambler should bet $f^* = 10\%$ of the existing funds in each game to maximize the long-term growth rate of the funds:

$$\Delta \ln N = 0.4\ln(1 + 2 \times 0.1) + 0.6\ln(1 - 0.1) = 0.97\% \tag{4}$$

In addition to maximizing long-term growth, the Kelly formula does not allow the possibility of losing all existing capital in any bet, so there is no fear of bankruptcy. The formula assumes that money can be divisible indefinitely, but as long as there is enough money, it is not a problem in practical application. The Kelly formula gives venture capital strategy is more rational. If the investment ratio $f = 20\%$. If f^* is too high, the long-term growth rate $\Delta \ln N = 0.07\%$ is obtained by recalculating the above equation, which is far less than the result of Kelly's formula.

3.2. Applications of Kelly's formula

This section provides an overview of the different applications of growth-optimal methods that have been developed using the discoveries discussed in the previous section. The primary areas of focus in our study encompass portfolio selection and asset pricing, risk management, and investment. The modeling of capital markets utilizing logarithmic investor utility, as well as the utilization of growth-optimal portfolios as numerical portfolios, have been fundamental approaches in portfolio selection and asset pricing. These methodologies have been employed both historically and in contemporary times to address intricate asset pricing issues. Risk management involves the study of risk in the form of fluctuations in the path of wealth and final wealth exhibited by growth-optimal strategies, as well as the risk of investor insolvency that comes with avoiding maximization of the expected geometric growth rate. Investment, on the other hand, pertains to more practical applications such as gambling and investing. From a mathematical perspective, gambling and investing are essentially the same. For instance, using a card-counting strategy in blackjack can be regarded as exploiting questionable stock market anomalies.

3.2.1. Portfolio selection and asset pricing

Ongoing research has been conducted on the utilization of growth-optimal portfolios in the field of asset pricing. Significant contributions encompass the utilization of numerical portfolios in the domains of derivatives pricing, risk management, and portfolio optimization. Long established the

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notion of numerical portfolios, wherein the valuation of all assets is determined in relation to one another. The author differentiates between numerical portfolios and growth-optimal portfolios, with the aim of simplifying the representation of asset values by expressing them in relation to the numerical portfolio. It should be noted that this approach may only be applicable to a restricted range of assets. In instances of this nature, it is not guaranteed that the numerical portfolio will be growth-optimal in comparison to the whole market [4].

Bajeux-Besnainou and Portrait provide a review of the literature on digital portfolios, portfolio properties, and their extensive links to continuous-time versions of multiple capital market theories [5]. Platen extends Long's work and develops a framework for pricing assets based on a numerical approach. Platen uses a growth-optimal portfolio as a numerical portfolio of asset prices and finds that the portfolio is a combination of a market portfolio and a savings account [6].

Du and Platen utilize Platen's benchmark approach based on a growth-optimal portfolio to manage risk in the form of incompletely replicable contingent claims [7]. Platen and Rendek approximate growth-optimal portfolios through plain diversification and show that their approximation performs better and is more stable over time than the sample-based Markowitz mean-variance approach, which is highly sensitive to changes in estimates over time [8].

Baldeaux et al. use Platen's benchmark approach to price long-term currency derivatives. They find that under their stochastic volatility model, the risk-neutral approach fails when calibrated to real data, while the benchmark approach performs better. Based on this result, they conclude that a long-term approach using a risk-neutral pricing paradigm may be particularly inappropriate for long-term derivatives [9].

Current research on growth-optimal methods and their correlation with digital portfolios suggests that the investigation into the implementation of growth-optimal portfolios in asset pricing and portfolio selection remains highly active, comparable to the level of activity observed following the initial publications by Kelly and Latané [10]. It is noteworthy to acknowledge that a shift in attitude has been observed in contemporary scholarly publications. The existing body of research does not put forth the notion that the growth-optimal model possesses a theoretical superiority over Markowitz's portfolio theory. The objective of this endeavor is not to exceed the constraints of portfolio theory by introducing a novel model. Furthermore, the analysis underscores the congruence between the two models and the efficacy of the growth-optimal portfolio as measured by the mean squared efficiency. The growth-optimal portfolios have the potential to function as benchmark risk/return portfolios or portfolios that are more easily accessible and stable within practical limitations compared to portfolio theory.

3.2.2. Risk and Kelly criteria

Latané's research conducted in 1959 suggests that growth-optimal techniques possess the capability to prevent insolvency, although displaying considerable fluctuation in final wealth. Previous research has demonstrated that exceeding the growth-optimal level of risk will result in heightened short-term volatility and diminished anticipated long-term growth, as evidenced by the seminal works of Breiman in 1961 [11], and Markowitz in 1976 [12]. Thorp's study conducted in 1984 also demonstrated that when the risk of optimal growth is doubled, it can lead to significant fluctuations in wealth around the original wealth level, without any anticipated long-term growth [13]. The precise nature of the relationship between the Kelly criterion and risk remains somewhat ambiguous. The Kelly method is deemed to have a certain level of risk and is deemed to be optimal solely under the premise of logarithmic utility in the context of final wealth, as per the anticipated utility theory.

MacLean et al. conducted a comprehensive simulation study to evaluate the long-term effectiveness of the Kelly strategy. They assessed the performance of the growth-optimal strategy by analyzing the probability of reaching different wealth targets and by considering the likelihood of

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experiencing losses in both full and partial Kelly wagers [14]. Rubinstein offers a practical approach for analyzing stock market investments based on a growth optimal strategy. According to Rubinstein, although the Kelly strategy accumulates more wealth in the long run, it requires a long-term perspective to outperform other strategies with high probability. The paper also explores other investment strategies, such as all-cash and all-equity strategies, and finds that it takes 208 years and 4,700 years with 95% certainty for them to outperform the Kelly strategy [15].

According to Ziemba's 2015 study, exceeding growth-optimal levels of risk-taking may endanger financial viability and reducelong-term growth, which is often the root cause of hedge fund catastrophes [16]. Therefore, using the Kelly criterion as a risk management tool may help prevent these catastrophes. However, it is essential to have a correct estimate of tail risk to enable the relevant use of the Kelly criterion. Flawed approximations of the probability distribution of outcomes or incorrect parameter estimates can distort the results, which may lead to excessive risk-taking that reduces long-term growth and potentially endangers financial survival.

3.2.3. Kelly strategies in gambling and investment

The accurate and optimal resolution of the capital growth problem might pose significant challenges or may even be unattainable due to practical limitations and the implementation of risk mitigation strategies. The authors Luo et al. presented a novel portfolio management model that utilized a simulated annealing technique for addressing the challenge of capital growth within the confines of real-world limitations. The authors proposed a risk management approach that incorporated several widely used risk measures, including bankruptcy risk, variation measure, and shortage measure [17]. Luo et al. extended the limited limitation and put out a broader application of the simulated annealing algorithm for the purpose of optimizing capital growth [18].

The study conducted by MacLean et al. investigated the phenomenon of capital growth within the framework of asset price dynamics that are contingent upon the prevailing regime. The researchers employed a geometric Brownian motion model inside each regime and imposed a value-at-risk constraint that must be satisfied at every time point. Additionally, they introduced a convex underperformance penalty for instances of non-compliance. This penalty not only ensured a reduced likelihood of experiencing losses but also imposed a punishment proportional to the severity of the losses suffered. The computation of optimal bets was performed deterministically at discrete time intervals. In their study, MacLean et al. discovered that the use of a convex penalty function resulted in the smoothing of the wealth path, thereby achieving an optimal equilibrium between the management of risk and the growth of capital [19].

Investigations into the behavior of investors have revealed that many prominent investors tend to concentrate their investments on a single stock, which often leads to superior long-term returns in the stock market. According to Ziemba 2005's study, the investment decisions and portfolios of these investors are more closely aligned with a full or partial Kelly strategy than any other investment approach [20]. Gergaud and Ziemba's study in 2012 have revisited this analysis with additional data and have confirmed Ziemba's findings in 2005, highlighting the high concentration of large investors in a single position and their consistent high performance [21]. These papers focus on the investment practices of John Maynard Keynes, who manages the King's College Endowment Fund, Warren Buffett and Berkshire Hathaway, and George, among others. Ziemba 2015's study further claims that the Kelly strategy is the only approach that can fully explain the investment behavior and performance of these investors.

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4. Some typical examples of the application of Kelly formula in gambling and investment

Taking the game example in literature review and the calculation process is (bp-q)/b = (2*50%-50%)/2 = 25%. From the formula, we can get a little inspiration for our investment:

Only when the edge (bp-q) is positive, the game can be bet. This is the most basic principle of all gambling and investment, that is, "never bet unless you are sure". You have to divide edge by odds b to get the invest ratio f. In other words, if the edges are equal, the smaller the odds, the more you can bet. These are three game assumptions:

Table 1: Three game assumptions

Game	p	b	(bp-q)	Kelly's optimal solution
1	20%	5	20%	4%
2	60%	1	20%	20%
3	80%	0.5	20%	40%

From the figure above, using Kelly's formula, we know that the "low win rate but high odds" game can only be bet 4% of the total money, but most people would probably choose the "low win rate and high odds" game, and they might spend all money. However, the rational choice would be "high win rate and low odds", because it can use 40% of the position. It is much faster than the others.Therefore, when we invest in stocks, if we want to increase our short-term position, perhaps the best choice is to consider a heavy position in large-cap stocks with low volatility but a high probability of rising, while for small-cap stocks with severe volatility, we should keep a low position operation. Let's assume that when the market is good:

Table 2: Market conditions when the market is good

Suppose that 40% stop profit and 10% stop loss. (b=40%/10%=400%)			
b	p	q	Kelly's optimal solution
400%	10%	90%	-13%
400%	20%	80%	0%
400%	30%	70%	13%
400%	40%	60%	25%
400%	50%	50%	38%
400%	60%	40%	50%
400%	70%	30%	63%
400%	80%	20%	75%
400%	90%	10%	88%
400%	100%	0%	100%

The Kelly formula above states that if the assumptions are satisfied, you can sell with a 30% certainty when the market is good.

Let's assume that when the market is bad:

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Table 3: Market conditions when the market is bad

Suppose that 3% stop profit and 10% stop loss. (b=3%/10%=30%)			
b	p	q	Kelly's optimal solution
30%	10%	90%	-290%
30%	20%	80%	-247%
30%	30%	70%	-203%
30%	40%	60%	-160%
30%	50%	50%	-117%
30%	60%	40%	-73%
30%	70%	30%	-30%
30%	80%	20%	13%
30%	90%	10%	57%
30%	100%	0%	100%

Kelly’s formula illustrates that in a bad market unless you are 80% confident of winning, you should not take any action. If the above formula seems a bit complicated, consider Buffett's version of Kelly's formula:

$$f = 2p - 1$$

Where: f= percent of investor’s capital to put into a single trade; p= probability of winning.

Taking the above case as an example, if the market is poor and there is an investment opportunity with an 80% probability of making a profit, then buy the stock position of $2 * 80\% - 1 = 60\%$. If there is a 100% profit-making investment opportunity, then buy all the stock positions. So Buffett's formula is simpler, but it seems more aggressive than the original because it ignores the odds. Buffett once advised that when gold falls to the floor, remember to grab it with a bucket, not a thimble. In fact, Buffett not only suggested it, he did it. When Warren Buffett bought Coca-Cola in 1988, he put a third of his money into it. He invested 40 percent of the partnership's net worth in American Express. These investments are the classic application of the Kelly formula to investing.

The Kelly formula serves as both a strategy for asset allocation and a safeguard against risk. The Kelly formula can be employed to mimic a given scenario. Benny currently possesses an initial capital of 100 yuan. He intends to engage in a series of four coin flips. In the event that the coin lands on heads, Benny will receive a return of capital amounting to six times the initial investment ($b = 5$). Conversely, if the coin lands on tails, Benny would incur a loss equivalent to the initial capital.

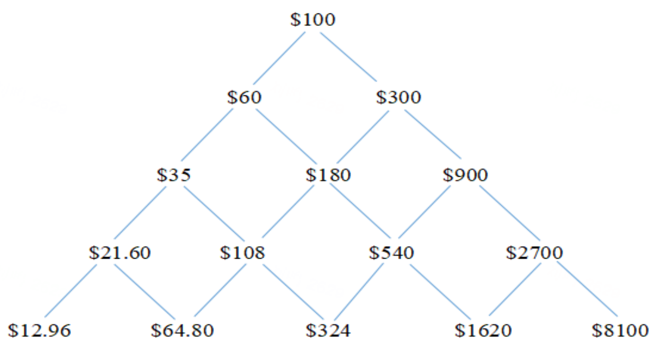

Figure 1: 4 Pascal’s triangle

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According to the calculation of Kelly's formula, we could calculate the best position is 40%. It can be seen that among 16 possible results (4 throws), 12.96 and 8100 appear once. 64.8 and 1620 occur 4 times, and 324 occurs 6 times. The purpose of Kelly's formula is to maximize the benefits of these outcomes. Since the Kelly formula focuses on long-term returns and risk control, investors naturally want to apply it to their investments.

5. Some critiques of Kelly's criterion

In the previous discussion, we established that the Kelly criterion and growth-optimal strategies are based on logarithmic utility. However, it should be noted that pursuing a fully Kelly strategy would not be the optimal choice for investors whose utility function differs from logarithmic. Although in the long run, the wealth accumulation from a full Kelly strategy outperforms any other strategy, it is not utility-maximizing for non-logarithmic utility functions. It may seem obvious that full Kelly is only optimal for logarithmic utility, but this concept has sparked intense debate in numerous publications, as has the asymptotic property of growth-optimal strategies. Early advocates of log utility-based financial market models either assumed that log utility approximates all investor utility functions or relegated utility theory to a secondary position. However, Rubinstein emphasized that models that restrict utility functions are preferred over those that restrict return distributions [22]. Moreover, proponents of anticipated utility theory expressed skepticism towards Latan'e's suggestion of growth-optimal strategies as a potential alternative to expected utility maximization. According to Samuelson [23], log utility is deemed to be a suboptimal approximation of individual risk preferences, irrespective of its computational efficiency or its asymptotic properties in the long term. Contemporary advocates of growth-optimal strategies recognize the mathematical constraints of previous studies and the theoretical criticisms against the extension of log-utility by Samuelson. However, they refute the underlying inference that growth-optimal tactics are typically devoid of value. MacLean et al. and Ziemba argue that the exploration of growth-optimal techniques has been limited due to Samuelson's resistance and the deficiencies found in the works of prior scholars.

The limitations of the Kelly formula therefore also deserve our attention, especially in the field of investment. Firstly the odds of winning p and the odds of winning b are subjective. The values of p and b perceived by each investor of the same target may be different, and optimistic investors will estimate higher win rates and odds. Next, win p and odds b cannot be measured precisely. Back testing using historical data is an option, but it is hard to believe. This is because history is not the future and the future is unknowable. Furthermore, both the winning rate p and the odds b are dynamic changes in an investment, and unexpected events can dramatically change the winning rate and odds. The investment version of the Kelly formula cannot withstand a black swan event. This is very different from a casino, where the casino owner limits the maximum odds at the table, but the market is different because as the odds become more favorable, the investment version of the Kelly formula gains an increasingly large percentage of the investment and can easily explode when a black swan event occurs, especially if the investor increases leverage. Even with prudent money management, it lost 65% in 1987. Failure to take into account the correlation between multiple investments in a portfolio, where multiple targets in the market are interconnected, means that the investor's risk is further increased. The accuracy of the win rate p is therefore an assessment of the investor's level of performance even if the odds ratio b is clear.

6. Conclusion

This paper presents a thorough examination of the historical context around the Kelly criteria, the process of formulating the mathematical equation, and practical illustrations of methods that optimize growth. The focus is primarily on the utilization of these strategies in the realms of asset pricing, risk management, and capital growth concerns. Simultaneously, the present research posits that the Kelly formula offers a framework for conceptualizing the expeditious expansion of investment yields. The Kelly formula can be regarded as a cognitive framework and a reference instrument that facilitates the logical management of one's positions. It is advisable to maintain objectivity and realism when assessing one's chances of winning and the associated probabilities prior to applying the Kelly formula. When engaging in the process of placing a wager, it is advisable to record the probabilities of achieving a successful outcome in order to inform and influence one's stance. This approach can effectively mitigate the tendency to indiscriminately adopt positions without careful consideration. Undoubtedly, it is vital to acknowledge the inherent limitations of the Kelly formula, necessitating investors to further enhance their precision in assessing the probabilities of success.

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