

Enhancing Portfolio Allocation by LSTM Model

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Abstract: Accurate forecasting of stock prices and the construction of optimized portfolios are essential for investors in the dynamic technology sector. This paper proposes a comprehensive approach that combines Long Short-Term Memory (LSTM) models with portfolio construction techniques specifically tailored to the stocks of Apple Inc. (AAPL), Meta Platforms Inc. (META), Amazon.com Inc. (AMZN), Microsoft Corporation (MSFT), and NVIDIA Corporation (NVDA) from January 1st, 2018 to May 31st, 2023. By leveraging the sequential nature of historical stock price data (80% of data), LSTM models capture complex patterns and dependencies, enabling more precise predictions of Adjusted Close (Adj Close) prices. Subsequently, the forecasted prices (20% of data) are utilized to construct optimized portfolios that maximize returns and minimize risks within the technology sector using Monte Carlo simulations, efficient frontier analysis, and key risk-return metrics. The overall result of the prediction data is similar to the actual data which implies that the integration of LSTM-based forecasting and portfolio construction provides a robust framework for informed investment decision-making and risk management. And the application of Monte Carlo simulations, efficient frontier analysis, and key risk-return metrics gave us two portfolio allocation options: Minimum Variance model (40% of AAPL, 60% of MSFT) and Maximum Sharpe Ratio model (47% of META, 53% of NVDA). The evaluation of the two portfolios shows that the strategy can significantly beat the SP500 index.

Keywords: portfolio allocation, LSTM, SP500

1. Introduction

Portfolio optimization plays a critical role in achieving optimal risk-return trade-offs and maximizing investment performance [1-2]. Efficiently allocating investment resources and constructing well-diversified portfolios are key challenges in the field of finance [3].

Through a review of the existing literature, we find a scarcity of research combining machine learning and portfolio optimization [4-5]. This research gap highlights the importance and relevance of our study.

In this paper, we address the aforementioned research gap by investigating the specific problem and proposing novel approaches. Specifically, the empirical investigation is generalized by three steps. First, this paper forecasts the stock price by the LSTM method. Then, we implement portfolio optimization

by these forecasts. Finally, we evaluate the performance of the portfolio by actual return of assets, and the results show that our model outperform the market.

This paper is structured as follows, Data section introduces stocks chose and data in the study. Methods section describes means of Monte Carlo and LSTM. Results and the conclusions will be presented in Section 4 and Section 5. Through our research efforts, we aim to contribute to the existing knowledge on portfolio optimization and provide valuable insights for investors and financial practitioners.

2. Data

This paper selects 5 representative technology company stocks (See Table 1). The ticker of the 5 stocks is AAPL, META, AMZN, MSFT, NVDA. Closing prices from January 1st, 2018, to May 31st, 2023, are collected from Yahoo finance (<https://finance.yahoo.com/>) and separated into training set (80% of data) and test set (20% of data). Finally, 1050 data are collected.

Table 1: Selected stocks.

Stock Symbol	Company
AAPL	Apple Inc.
META	Meta Platforms Inc.
AMZN	Amazon.com Inc.
MSFT	Microsoft Corporation
NVIDIA	NVIDIA Corporation

We transfer these closing prices to Compounded return (log-return) and calculate some basic information shown in the following Table 2:

Table 2: Descriptive statistics of five stocks.

	'AAPL'	'META'	'AMZN'	'MSFT'	'NVIDIA'
Mean	0.00108	0.000272	0.000527	0.001040	0.001541
Std	0.020673	0.028219	0.022625	0.019480	0.033168
Skew	-0.234094	-1.592374	-0.167771	-0.231357	-0.215494
Kurt	4.743682	23.490644	3.939795	6.798225	4.648209

Also, we remove the outliers and get cumulative returns of those five stocks in Figure 1.

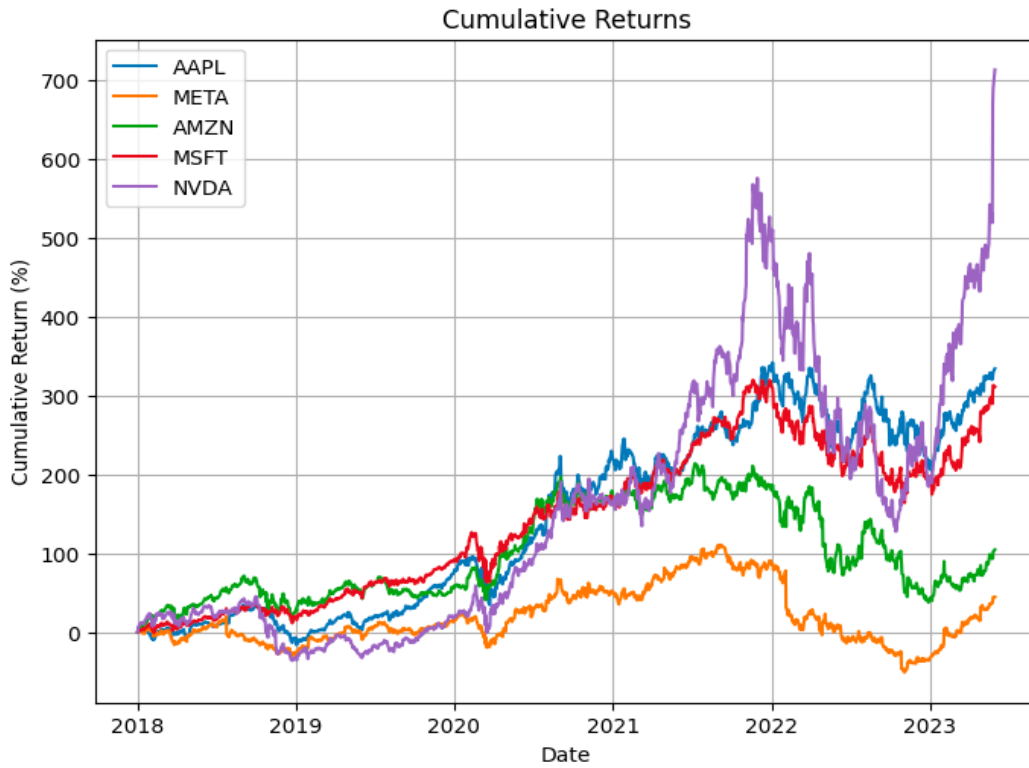


Figure 1: Cumulative returns of selected stocks.

3. Methods

3.1. Monte Carlo Model

Monte Carlo model is employed to assess the performance and risk characteristics of the optimized portfolios. It generates a large number of hypothetical scenarios by randomizing the returns of each stock within their expected ranges. By simulating these scenarios repeatedly, the method can provide a distribution of potential outcomes [6].

3.2. Efficient Frontier

Efficient frontier analysis plays a crucial role in portfolio optimization, which identifies the set of portfolios that offer the highest returns for a given level of risk. The equations are expressed as follows:

$$\text{Maximize } \mu \tag{1}$$

$$w_1 + w_2 + \dots + w_n = 1 \tag{2}$$

$$\sigma^2 = w^T \Sigma w \tag{3}$$

$$\mu = w^T \mu \tag{4}$$

where μ is the vector of expected returns, $w = [w_1, w_2, \dots, w_n]$ represents the vector of portfolio weights, Σ is the covariance matrix of the selected stocks. By solving this optimization problem for different levels of risk, and analyzing the expected returns, volatilities, and correlations among the selected stocks, we can obtain a range of portfolios that lie on the efficient frontier [7].

3.3. Sharpe Ratio

The Sharpe ratio is a widely used measure of risk-adjusted performance in portfolio management. The Sharpe ratio portfolio model aims to construct portfolios that maximize the Sharpe ratio, indicating higher risk-adjusted returns. By considering the expected returns, volatilities, and correlations of the selected stocks, it determines the optimal allocation of weights to each stock in the portfolio.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \quad (5)$$

Where R_p is the portfolio's expected return, R_f is the risk-free rate of return, σ_p is the portfolio's standard deviation (volatility). A higher Sharpe ratio indicates a better risk-adjusted performance, as it implies a higher excess return per unit of risk.

3.4. LSTM

Long Short-Term Memory (LSTM) can capture long-term dependencies and handle irregular time series data with high predictive accuracy. LSTM is a variant of recurrent neural network (RNN). It solves the Vanishing gradient problem in traditional RNN by using gating unit structure [8-10] (See Figure 2).

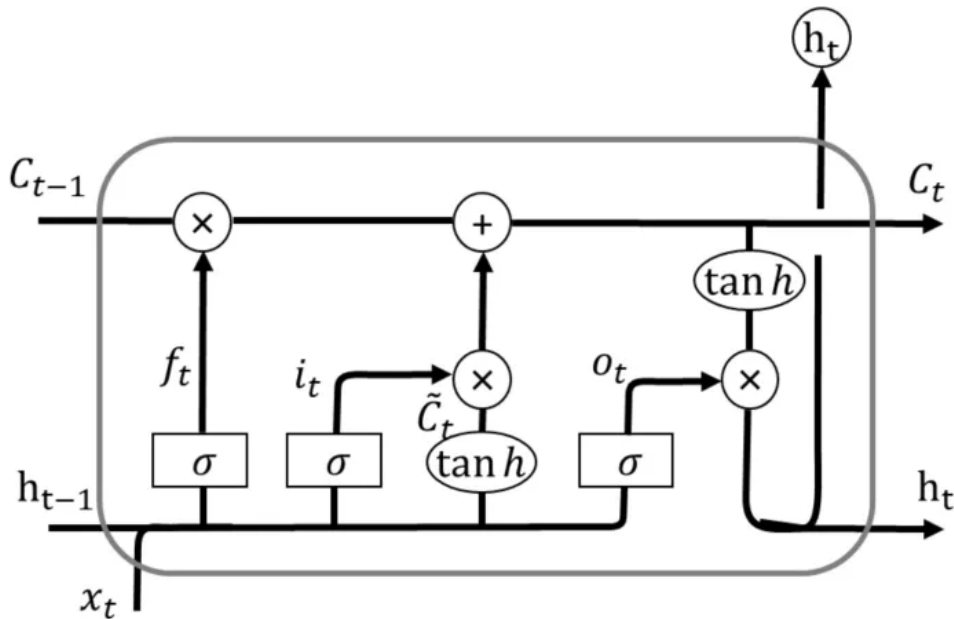


Figure 2: Cell Structure.

LSTM is formed by repeated module chains, with each individual module referred to as a cell. Each cell has a specific gate structure to selectively transmit information and updates the state of each cell are achieved through the transfer of information through LSTM gate structures (forget gate, input gate, hidden state, output gate). The structure of the cell is shown in the figure. Where σ is the Sigmoid Activation function, \otimes is the Hadamard, and \tanh is the tanh Activation function.

The function of the forget gate is to selectively update the cell state. The first step of LSTM is to determine the update of the cell state. It processes h_{t-1} (previous output) and x_t (current input)

through the sigmoid function, outputting numbers between 0 and 1. 1 represents complete retention, while 0 represents complete forgetting. Where w_f is the weight matrix and b_f is the bias term.

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \quad (6)$$

The function of an input gate is to input data and determine the information stored in the cell state. This section is divided into two steps. Firstly, the Sigmoid layer determines the weight of input information in the update, which information needs to be updated. Next, we use tanh to construct the candidate vector \tilde{C}_t calculated at this time.

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_f) \quad (7)$$

$$\tilde{C}_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c) \quad (8)$$

The final output gate will output based on the current cell state, passing the current input data through the sigmoid layer. Then, the current cell state is passed through the tanh layer, the matrix is normalized to between -1 and 1, and it is then subjected to a basic product operation with the output of the sigmoid layer. At this point, the data of the output part is determined.

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad (9)$$

The module chain network composed of LSTM cell units mentioned above can predict the future through continuous learning of input data.

$$h_t = o_t \cdot \tanh(C_t) \quad (10)$$

4. Result

Figure 3 demonstrates the comparison between real adjusted Close prices and forecasted adjusted close price of five stocks (AAPL, META, AMZN, MSFT, NVDA) from July 28th, 2022, to May 31st, 2023, which shows the small differences between the adjusted close price forecasted by LSTM model with the real data.

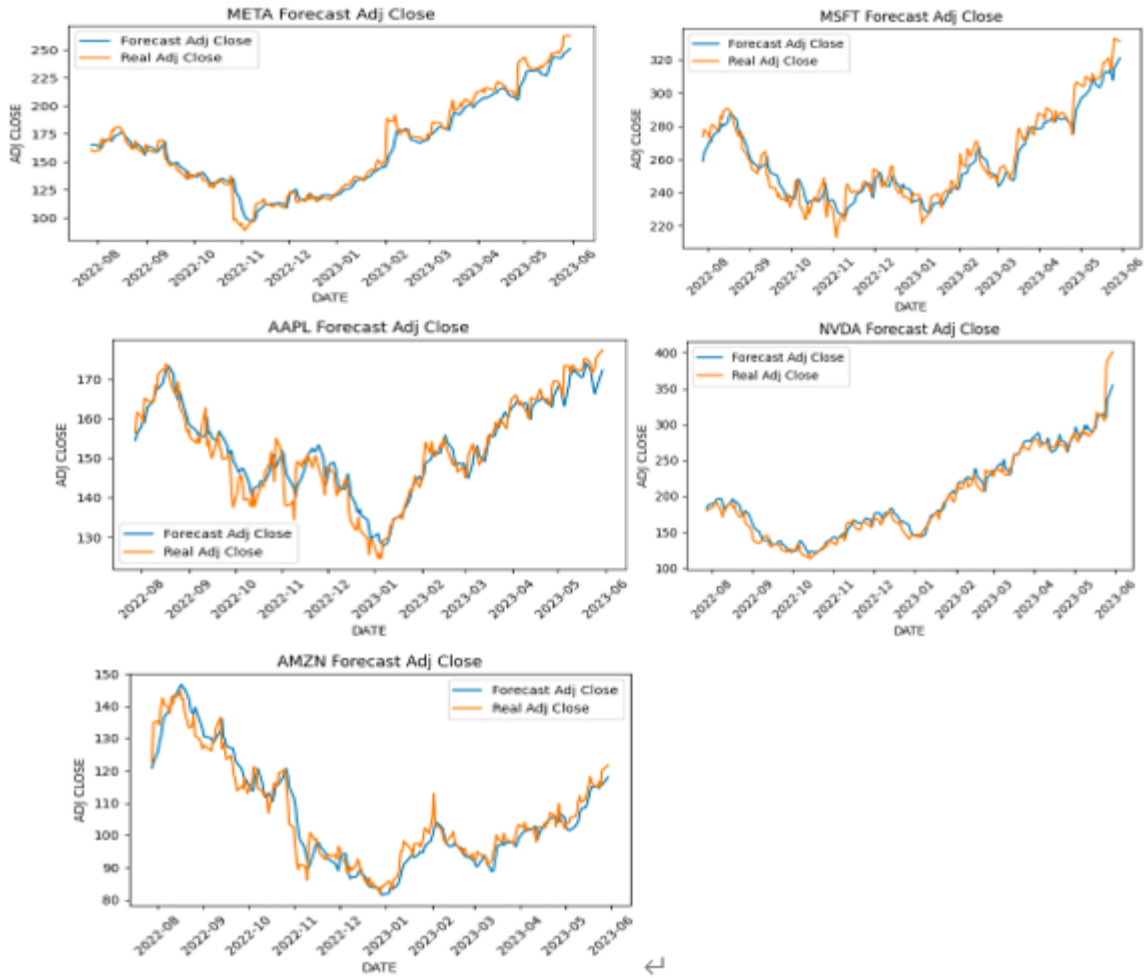


Figure 3: Predicted Data.

Secondly, this article uses the Monte Carlo model to assign 40,000 different portfolio weights to the results and shows the expected returns and volatility of these different portfolios in the same graph shown in the following Figure 4:

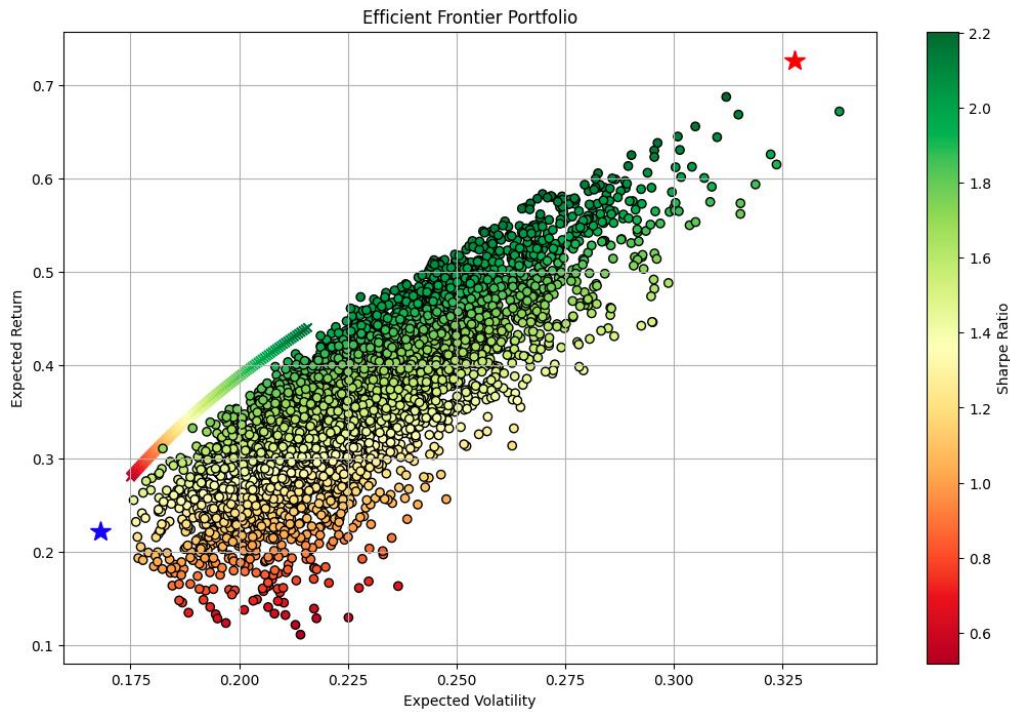


Figure 4: Monte Carlo Graph.

The circles in the Figure 4 represent the combination distribution randomly generated by Monte Carlo, the cross marks represent the efficient frontier, the red star marks the maximum Sharpe ratio combination, and the blue star marks the minimum variance combination. According to the portfolio investment model, we have two allocation options, the first is the maximum Sharpe ratio portfolio model, and the second is the variance minimum portfolio model (See Table 3).

Table 3: Asset weights.

Weight	AAPL	META	AMZN	MSFT	NVDA
Largest Sharpe Ratio	0	47%	0	0	53%
Smallest Variance	40%	0	0	60%	0

Based on these asset weights, this paper evaluates the portfolio performance and shows the results as follows in Figure 5:

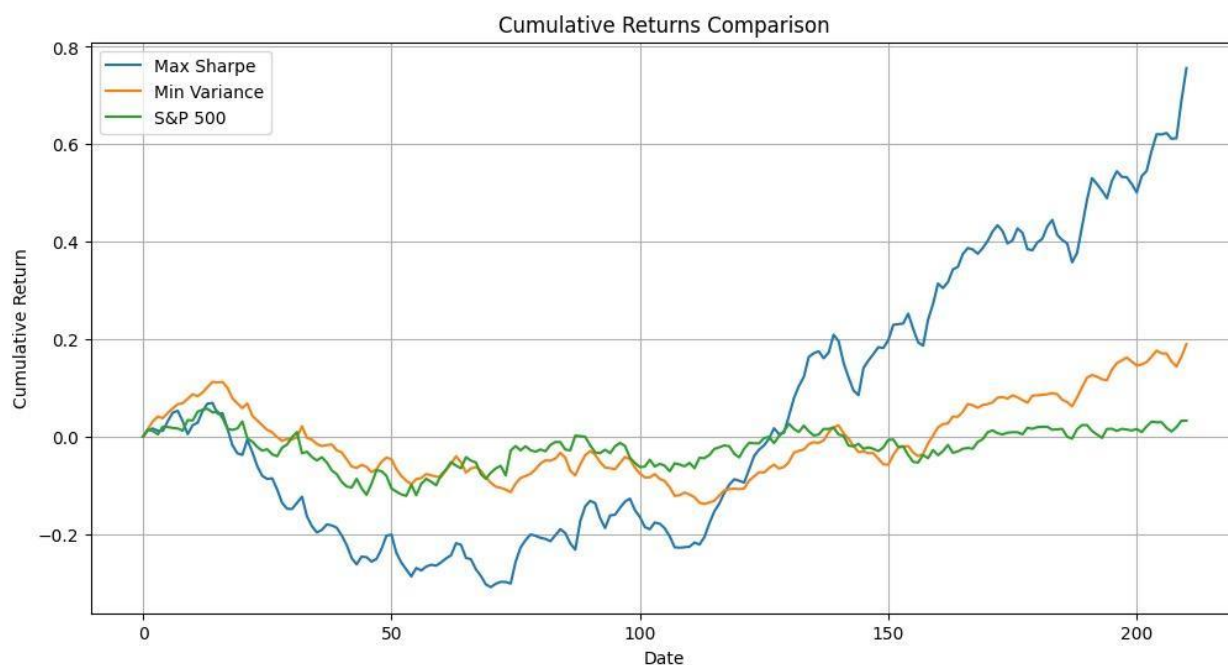


Figure 5: Cumulative Returns of two models and S&P 500 index.

The green line represents the S&P 500. At the first 50 days, minimum variance model has the highest cumulative return. Between 50 and 100 days, S&P 500 has the highest portfolio return. After 100 days, the maximum sharpe model catches up with the biggest slope and surpasses the other two to possess the highest cumulative return, so as minimum variance model catches up with a smaller slope and surpasses the SP500.

5. Conclusion

This article combines the Long Short-Term Memory (LSTM) model with portfolio construction techniques. By utilizing the continuity of historical stock price data (80% of the data), the LSTM model captures complex patterns and dependencies, enabling more accurate prediction of adjusted closing (Adj Close) prices. Subsequently, the forecast price (20% data) was used to build an optimized portfolio, using Monte Carlo simulation, effective frontier analysis and key risk return indicators to maximize the return of the technology sector and minimize risk. The application of Monte Carlo simulation, effective frontier analysis and key risk return indicators provides us with two portfolio allocation options. In 200 test days ranging from 50 to 100 days, the S&P 500 index has the highest return on investment portfolios. After 100 days, the maximum sharp model caught up with the maximum slope and surpassed the other two, resulting in the highest cumulative return, while the minimum variance model caught up with the smaller slope and exceeded SP500. However, it is important to note that these results are based on historical data and assumptions, and future market conditions may impact the performance of the optimized portfolio. Thus, investors should regularly monitor and adjust their portfolios to adapt to changing market dynamics and ensure their investment objectives are met.

Authors Contribution

All the authors contributed equally, and their names were listed in alphabetical order.

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