Proceedings of the 3rd International Conference on Business and Policy Studies
DOI: 10.54254/2754-1169/75/20241602

An Integrative Review of Portfolio Management

Yuechuan Zhao[1,a,*]

[1]School of Mathematical Sciences, Nankai University, Tianjin, 300071, China
a. 2012195@mail.nankai.edu.cn
**corresponding author*

Abstract: This paper delves into the evolution of portfolio theories, tracing their development from the seminal work of Markowitz's traditional portfolio theory to the contemporary landscape of modern portfolio management. Emphasizing the relevance for real-world investors, the study scrutinizes the advantages, applications, and limitations of these investment theories. The exploration commences with a comprehensive analysis of return, risk, and various factors integral to portfolio theory. Leveraging qualitative analysis and real-world examples, the paper elucidates the mathematical model underpinning the theory, furnishing investors with a structured investment strategy. The focal points of the paper are the VaR (Value at Risk) model and the beta coefficient. Through an in-depth examination of their mathematical foundations and theoretical underpinnings, the research elucidates the symbiotic relationship between their economic significance and mathematical intricacies. Furthermore, the paper expounds on their collective contribution to the overarching portfolio theory, shedding light on their role as guiding principles for risk-averse investors. By weaving together theoretical frameworks and practical implications, this paper seeks to provide a nuanced understanding of portfolio theories, equipping investors with valuable insights for informed decision-making in the dynamic landscape of financial markets.

Keywords: Portfolio Management, Asset Allocation, Diversification

1. Introduction

Before discussing the relevant contents of portfolio management, the background of the development of investment theory needs to be discussed to some extent. In Markowitz portfolio theory before the molding, people's discussion of the portfolio remains in a relatively superficial stage, although there has been a diversified investment to avoid risk awareness, and not a full theoretical basis for the control of risk and profit is not very rational but stays in the simple investment dispersed in different industries, different geographic areas, different markets, etc., the specific dispersal method Lack of quantitative analysis.

By 1935, Hick put forward the principle of separation, its content mainly focuses on the diversification of investment to diversify the risk of the point of view, but also emphasized that due to the pursuit of high yield and low risk, there is always a need for investors to invest in the performance of the investor, which makes the construction of a "theory of money" is very necessary, and will introduce risk into the analysis. analysis. In addition, the concept of risk compensation was also proposed in this period, which explains that due to the existence of uncertainty, financial assets always need to give certain compensation to their investors.

DOI: 10.54254/2754-1169/75/20241602

In 1938, the "diversification model" pointed out that as long as the investment in a sufficient number of financial assets, you can offset the risk of uncertainty, which implied the existence of a portfolio to meet the maximum return and minimum risk, which laid the theoretical foundation for the establishment of Markowitz's portfolio theory. Subsequently, scholars argued the benefits of diversification.

In 1952, Markowitz, an American economist, concluded in his thesis that risk can be effectively reduced through investment portfolios. This analysis by the mean-variance analysis model marked the beginning of modern portfolio theory, and at the same time, the effective boundary model of the portfolio given by him demonstrated the revolutionary role of the scientific method in the field of investment portfolios, which also led to the exploration of later scholars, including the "two-fund separation model This also led to the exploration of later scholars, including the "two-fund separation model" and the "security prime model".

After that, the expansion of the portfolio mainly focused on the assessment of risk, unlike the variance in the traditional model, in the "mean - lower half variance model", the lower half variance instead of variance for analysis. At the same time, the Unconventional Yield Capital Asset Pricing Model (UYCAPM), which analyzes portfolio performance based on the security market line in the CAPM, was proposed, and the Arbitrage Pricing Theory (APT) was proposed later due to the overly stringent assumptions of the conventional model. After this, the B-S model was proposed to deepen the theory even further.

Throughout the development, It can be found that the idea has shifted from the traditional and simple massive investment in different assets to the quantitative analysis of the portfolio according to its risk and return from the correlation of different assets, which not only points out the existence of a theoretically optimal portfolio, but also explains that when the number of assets reaches a certain number, the management cost rises with it, and its risk aversion becomes less and less effective.

Next, the article will describe the development and theoretical logic of traditional Markowitz portfolio theory and analyze its rationality. It will then discuss its application in real-life scenarios. Finally, the article will discuss its limitations in the current form of the economy.

2. Objectives & Strategies

After a brief overview of the development of portfolios, it can be observed that the goal of modern portfolios is to find a specific combination of financial assets that makes it possible to maximize the benefits while taking the minimum amount of risk, and in order for the results to be quantified, the risks and returns need to be interpreted quantitatively, which implies that a number of mathematical concepts need to be introduced into the modeling and analysis.

The first thing that is discussed, is how to describe the returns in clear mathematical terms. Due to the uncertainty of the future, no one can give the future returns of any portfolio. Therefore, the description of returns can only be estimated based on past data, which requires the intervention of statistical tools to estimate returns by listing possible returns and giving their likelihoods, from which weighting operations are performed. These possible returns are often derived from statistics on past returns of financial assets; analysis of recent business operations of firms; and the ecology of the corresponding industry.

There is a difference between the expected returns characterized by expectations and the actual returns, and the underlying reason for this is the incomplete adequacy of market information; investors cannot get all the information about financial assets from the market, and the most powerful statistical tools cannot compensate for the lack of data, so even if the investor takes all the influencing factors into account in the analysis, the

According to the previous section, the Markowitz portfolio model uses variance to measure overall investment risk, this is because variance in the statistical sense reflects how much the sample deviates from the mean. When this concept is applied to financial assets, it can be interpreted as the extent to which a portfolio of multiple securities deviates from the whole, which reflects the "important effect of correlation between securities on portfolio risk" [1]. For the whole, the variance is generally a weighted sum of the variances of the individual assets.

In addition to the above, so-called diversification also requires attention to the correlation between the assets taken. For the sake of risk aversion, the investor of course hopes that even if one of the selected assets depreciates, the rest of the assets will not depreciate at the same time for the same reason. Therefore, in terms of mathematical concepts, the introduction of covariance enables the measurement of the relationship between two assets, a concept that refers to the difference between the expectation of the product of two sets of data and the product of the expectation, reflecting the independence of the different data, and when the covariance is zero, it means that the two sets of data are completely independent.

In this way, by constructing a set of functions, the investor is able to obtain a theoretically optimal curve, which scholars call the efficient frontier curve, which characterizes a portfolio with the highest return that can be found given a given level of risk. As the investor's acceptable risk varies, the points on the curve correspond to the optimal returns for the corresponding risk.

3. Risk Measurement

Based on the previous brief introduction, investors will note that simple linking equations are used here to decipher risk and return, but the fact is that economic operations are quite complex and stochastic, and simply treating them as a model consisting of multiple independent variables is not supported by the principles of economics. The complexity of economic operations makes the link between different variables may not be intuitive, which makes it difficult to accurately capture the correlation in the traditional way of research, i.e., the value of the dependent variable (independent variable) takes place one period before the change in the dependent variable (dependent variable). The treatment with a simple system of simultaneous equations can be extremely arithmetic intensive due to the fact that it is necessary to analyze the effects brought about by the lagged variable of the independent variable. This is reflected in determining the dependent and lagged variables.

Therefore, economists introduced the var model in 1980, which treats all variables as endogenous and can be regarded as the associative form of the regression model, which allows the investor to be informed of the relationship between the two variables of interest. The structure of the var model is mainly related to two parameters, namely the number of variables included, N, and the maximum lagged order, k. The former is well understood, while the interpretation of the latter requires the knowledge of the self-regression model, which is generally considered to be the most important factor to be taken into account. It is generally believed that in an autoregressive model, the value of a particular moment point is a linear combination of the values of previous moment points, and the coefficient of each parameter of this linear combination is the lag coefficient, and in general, the maximum lag order refers to the order corresponding to the maximum lag coefficient of the whole combination, which can help investors to select the appropriate model for estimation.

According to the above theories, investors can discover the correlation between different assets through the VaR model, and then determine their own investment strategies. Li concluded that Alibaba has a certain unidirectional guiding effect on the Shanghai Index, and its analysis of stock characteristics also shows that Alibaba has a certain unidirectional guiding effect on the Shanghai Index. Its analysis of stock characteristics also suggests that Alibaba's stock has a higher return than the Shanghai index and its ability to adjust to external shocks is stronger, which implies that Alibaba is superior to the Shanghai stock market in general as an investment option. And based on the historical data of the Chinese stock market, investors do get similar results [2]. For now, the VaR

DOI: 10.54254/2754-1169/75/20241602

method provides investors with a more convincing and relevant measure of market risk, and is now the dominant method of measuring market risk.

In addition to this, investors now need to focus some attention on the beta coefficient, which is generally used to measure the price volatility of individual financial assets relative to the stock market as a whole, and is a tool for assessing the systematic risk of a security, measuring the volatility of a security or a portfolio of investment securities relative to the overall market.

Before understanding this concept, investors need to make sense of systemic risk, which generally points to large-scale fluctuations that take effect in the face of the market as a whole, and this version includes changes in policy, the impact of interest rates and even the overvaluation or undervaluation of the market as a whole. This means that the impact of such risks can be felt across almost all financial assets, and therefore it is not possible for investors to avoid such risks through diversification. From there, investors can turn to beta coefficients to assess such risks, assuming that investors want to know the beta coefficient of a specific financial asset a versus a stock index b. Here, the data of Shunfeng Holding (002352.SZ) versus the CSI 300 Index (000300.SH) is used for the calculation:

Based on the website data, investors can calculate the variance of the daily return of the CSI 300 index: 1.56 (2019 data); the covariance of the daily return of the CSI 300 index (000300.SH) and Shunfeng Holdings (002352.SZ): 0.97 (2019 data). This leads to beta = covariance/variance. From the results, the beta coefficient reflects the volatility of a particular financial asset and the underlying asset to market movements. When beta is less than 1, the former can be considered less volatile than the latter, and the opposite is true when it is greater than 1 [3]. The reason why the beta coefficient can reflect systematic risk is that it essentially reflects the price volatility of a single financial asset versus the overall market (i.e., the underlying asset). On a portfolio, the overall beta coefficient is a weighted sum of the beta coefficients of the individual assets.

4. Asset Allocation & Diversification

In the previous section, investors will note that they can select portfolios based on their own risk considerations based on the effective marginal curve, where a mathematical representation of the classic Markowitz model can be given:

$$\max E(r) = X^T R \tag{1}$$

$$\min \sigma^2 = X^T C X \tag{2}$$

$$s.t. \sum x_i = 1 \tag{3}$$

Investors can easily see that this system of equations follows the principle of maximizing expected return and minimizing risk, although in reality investors would be hard pressed to find a portfolio that is so good as to maximize return while meeting minimum risk. Therefore, investors need to make a choice of objectives by simplifying the previous dual-objective planning to single-objective constrained planning, where the return or risk becomes a constraint based on their needs, typically setting the minimum expected return or the maximum risk [4].

In this way, the investor can solve the simplified system of equations to get a curve as shown below, which, gives all the portfolios that satisfy the investor's requirements.

However, in the configuration of the asset portfolio, the risk-free asset is also a key factor that cannot be ignored [5]. Based on its characteristics, the investor will find that the return of the portfolio is equal to the risky asset return and the risk-free asset return of the weighted sum of the return of the risky asset and the risk-free asset, and due to the risk-free asset's risk-free nature, its correlation

DOI: 10.54254/2754-1169/75/20241602

coefficient with the rest of the assets must be 0 [6]. As a result, the investor can simplify the elements in the portfolio for the two parts of the risky and the risk-free and get the two equations below:

$$R_p = w_{risk-free}R_{risk-free} + w_{risk}R_{risk} \tag{4}$$

$$\sigma_p = w_{risk}\sigma_{risk} \tag{5}$$

A simple conjunction allows the investor to obtain such an equation:

$$E(R_P) = R_{risk-free} + \frac{E(R_i) - R_{risk-free}}{\sigma_i}\sigma_p \tag{6}$$

This is the equation known to scholars as the asset allocation line (CAL) equation, and the CAL curve reflects the linear relationship between expected return and risk, in a sense giving the price of risk. There is an upper limit to the slope of the CAL curve, given the relatively fixed interest rate on risk-free assets and the efficient frontier already given [7]. Indeed, it is the point at which the CAL curve is tangent to the efficient frontier curve that investors should be concerned with, and to explain this, investors need to understand the undifferentiated demand curve [8].

The undifferentiated demand curve essentially reflects the degree of satisfaction that investors receive from different portfolios. Multiplying the expected return and variance of a portfolio by the coefficient of aversion and making the difference gives a mathematical sense of the degree of satisfaction of a portfolio for a given investor. If the degree of satisfaction is given as quantitative, an investor can obtain a curve based on his or her own aversion to risk. From this, assume that there exists a CAL(M) curve tangent to the efficient frontier curve with the largest slope, and if there is a CAL with a larger slope than CAL(M), at this point a tangent can be found to the undifferentiated curve, but not to the efficient frontier, which means that it is not possible to construct such a portfolio in practice [9].

If there is another CAL with a smaller slope than CAL(M), at this time, after finding the tangent point with the effective frontier, but cannot be tangent to the nondifferentiable curve, because at this time, multiple intersections are generated, but also above it can also find the tangent point, then there is no optimal combination to satisfy the conditions based on the investor's nondifferentiable curve [10].

5. Conclusion

By the end of the article, investors may find that existing portfolio theories do not necessarily deliver maximum returns or provide a foolproof investment strategy. The reasons here are quite varied and complex, a few important ones being that the information investors need comes from the market and the analysis is based exclusively on mathematical models.

Analysis based on historical information can certainly reflect a certain picture of financial assets and simplify the process for investors to derive data such as returns, variances and covariances. However, it is important to note that markets are not fully efficient, which means that the information available to the investor is incomplete, and that historical information may not always be a fully accurate reflection of current or even future price movements of financial assets. This means that the returns generated by the portfolios constructed by investors do not necessarily correspond to the calculated results, and in fact, small differences in the raw data may result in very different investment strategies. As a result, investors relying on historical data are not perfectly optimized, and the complexity of the mathematical process amplifies the effect of initial errors, suggesting that there may be cases where investors are guided by theory and instead obtain portfolios with losses.

DOI: 10.54254/2754-1169/75/20241602

While an analysis based entirely on mathematics is certainly rational and objective enough, and the processing of data and the derivation of conclusions is convincing enough, according to behavioral economics, investors cannot always be objective when faced with a variety of investment scenarios, including not only the influence of personal tendencies on investments, but also logic such as: frame shifting, loss aversion, and the obsession with small probabilities. This implies that although modern portfolio theory is sufficient to give a sufficiently rational investment strategy, it does not take into account the investor's personal inclinations, blurring the impact of psychological expectations on investment strategy.

References

[1] *Mei, T. (2010). Risk Measurement and Portfolio Management Based on VaR, Shanghai Jiao Tong University.*

[2] *Li, S. Y. (2021). An Empirical Research on the Relevance between Shanghai Stock Index and Alibaba—Based on VAR Model. Operations Research and Fuzziology, 11(03), 347–355.*

[3] *Financial Center. (2023). China Net. https://finance.china.com.cn*

[4] *Barna, T., Von Neumann, J., & Morgenstern, O. (1947). Theory of Games and Economic Behavior. The Review of Economics and Statistics.*

[5] *Arrow, K. J. (1964). The Role of Securities in the Optimal Allocation of Risk-Bearing. The Review of Economic Studies, 31(2), 91.*

[6] *Jorion, P. (1996). Risk2: Measuring the Risk in Value at Risk. Financial Analysts Journal, 52(6), 47–56.*

[7] *J.P.Morgan and Reuters. (1996). RiskMetrics—Technical Document Fourth Edition.*

[8] *Pratt, J. W. (1964). Risk Aversion in the Small and in the Large. Econometrica, 32(1/2), 122.*

[9] *DaníElsson, J., & De Vries, C. G. (1997). Beyond the Sample: Extreme Quantile and Probability Estimation. RePEc: Research Papers in Economics.*

[10] *DaníElsson, J. (1997). Extreme Returns, Tail Estimation, and Value-at-Risk. RePEc: Research Papers in Economics.*