

Portfolio Optimization Based on the LSTM Forecasting Model

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Abstract: The prediction of stock performance is a crucial component in formulating investment portfolios and optimizing portfolios within the realm of quantitative trading. However, the inherent unpredictability and volatility of the stock market pose significant obstacles for investors in accurately predicting stock performance. To build an optimal portfolio, the LSTM model is selected as a forecasting technique. Subsequently, data sourced from Yahoo Finance is acquired for training and testing purposes. Based on the prediction data, the paper applies the maximum Sharpe ratio model and the minimum variance model to reach portfolio optimization. Finally, the paper uses the S&P 500 index as a standard to evaluate the constructed portfolio. The results indicate that the LSTM prediction model has effective functionality and exhibits superior performance in the domain of data forecasting. In addition, the minimum-variance optimization and the maximum Sharpe ratio models explore optimized return and minimized risk in portfolio construction. The constructed portfolio outperforms the S&P 500 in terms of risks and returns. Therefore, the results in the paper are good for investors to reduce risk and increase return in portfolio construction.

Keywords: portfolio optimization, LSTM, investors

1. Introduction

A portfolio refers to a collection of various financial assets, including stocks, funds, bonds, real estate, and derivatives. As portfolio management is the art of selecting and forecasting the assets of investments, the aim is to adhere to the desired financial goals and minimize the risk tolerance of individuals or institutions. However, there are many factors involved in building and maintaining a high-return portfolio, including diversification, asset allocation, and especially the fluctuation of the stock market. The instability of the market attracts an increasing number of researchers and investors to pursue their passions in predicting stock prices to achieve high portfolio returns.

Portfolio construction is a big challenge for institutions and individual investors, primarily driven by the pursuit of higher returns. Therefore, researchers integrate portfolio construction models with forecasting models in order to investigate efficient portfolio allocations. The time series model is introduced to improve portfolio performance. For instance, Soeryana, Fadhlina, and Rusyaman took a time series approach to optimize the Mean-Variance portfolio [1]; Neto, Calvalcanti and Ren used

data extracted from exogenous series to do a time series forecast [2]; Zhou, Yang and Yu achieved portfolio optimization under the fuzzy time series method [3]. In recent years, there has been a surge in the utilization of machine learning and deep learning models for the purpose of portfolio development, such as random forest (RF), support vector regression (SVR), and long short-term memory (LSTM). In a study conducted by Ma, Han, and Wang [4], the authors employed machine learning and deep learning models to enhance return prediction in portfolio performance by integrating returns and risks in portfolio construction. Similarly, Ban, EL, and Lim addressed the limitations of mean-variance and mean-conditional value-at-risk models by incorporating performance-based regularization (PBR) into portfolio optimization [5]. Chen, Zhang, Mehlawat and Jia applied eXtreme Gradient Boosting (XGBoost) and the firefly algorithm (IFA) to predict stock prices and constructed a portfolio under the mean-variance model [6]. Moreover, Ma, Han, and Wang explored portfolio optimization by employing three deep neural networks: deep multilayer perceptron (DMLP), long short memory (LSTM), and convolutional neural network (CNN) [4]. However, the existing research on deep learning models and machine learning models is currently restricted and requires more practical implementation.

This paper combines the LSTM forecasting method for price forecasting to conduct portfolio construction and stock price prediction. The article initially identifies a set of 5 equities obtained from Yahoo Finance, spanning the time period from January 1, 2018, through December 31, 2022. Secondly, the paper adopts the LSTM model to train the first 80% of selected data and then tests the bottom 20% of selected data, yielding the predicted data. The Maximum Sharpe ratio and Minimum Variance portfolios can be computed based on predicted data and the mean-variance model. This study presents an analysis of cumulative returns derived from the Sharpe model and risk model depicted in a graph, as well as the cumulative returns of The S&P 500. By evaluating the Maximum Sharpe ratio and Minimum volatility portfolios, this paper can conclude that the portfolio constructed using predicted data has exceptional g performance compared to the S&P 500.

2. Data

For diversification purpose, the paper selected 5 stocks among various industries, which are Pegasystems Inc.; Wolfspeed, Inc.; Shell plc; Exxon Mobil Corporation; and DigitalOcean Holdings, Inc. The dataset comprises five years of closing prices, ranging from January 1, 2018, to December 31, 2022, which has been sourced from Yahoo Finance. The dataset is divided into a training set and a testing set, with the train-test ratio at 80:20. For further research, the paper processes the selected data.

Table 1: Selected stocks.

Stock Symbol	Company
PEGA	Pegasystems Inc
WOLF	Wolfspeed, Inc
SHEL	Shell plc
XOM	Exxon Mobil Corporation
DOCN	DigitalOcean Holdings, Inc

Based on selected database, the paper calculates several measurements and cumulative returns of 5 selected stocks during the 5-year period. Furthermore, the paper generates the cumulative returns of selected stocks. (See Table 2 and Figure 1).

Table 2: Statistics of selected stocks.

	PEGA	WOLF	SHEL	XOM	DOCN
Mean	86.5475	95.7838	48.3927	77.5748	53.4925
Std	39.0723	16.4390	6.6890	18.2654	20.6941
Skewness	-0.1405	0.2071	-0.0745	0.3449	1.3193
Kurtosis	-1.6201	-0.2800	-1.3926	-1.2155	1.5490

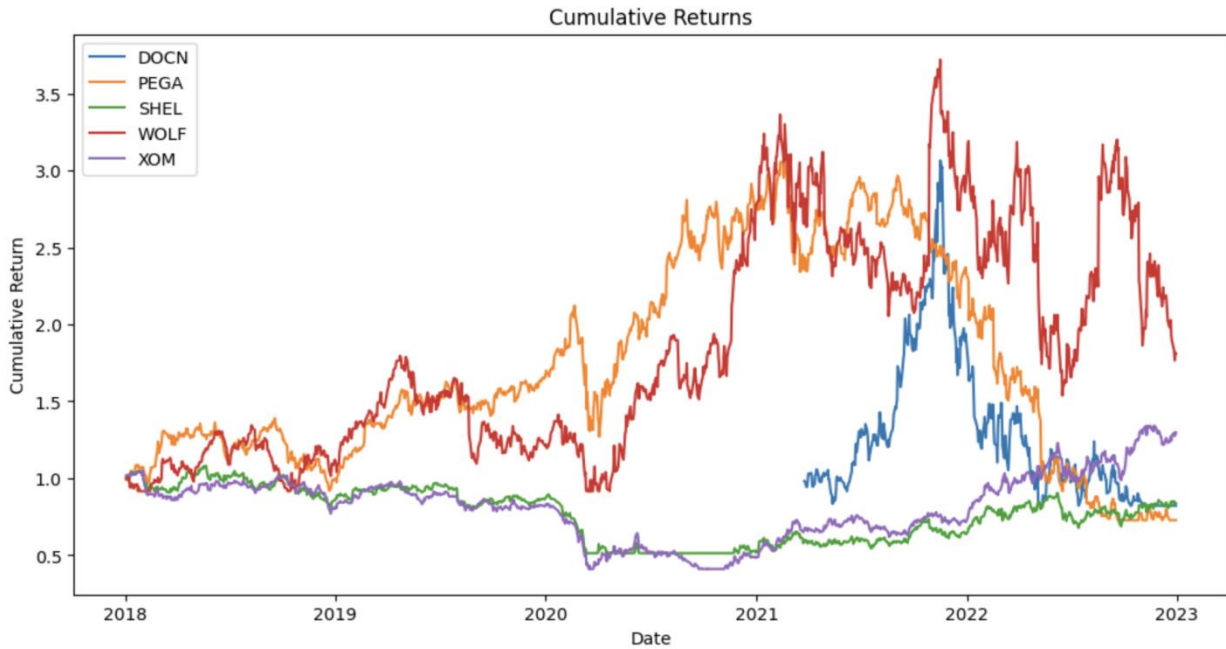


Figure 1: Cumulative returns of selected stocks.

3. Method

The portfolio allocation process consists of two distinct components. In the initial phase, data was gathered through Yahoo Finance. The utilization of Long Short-Term Memory (LSTM) prediction models has been observed in the domain of stock price forecasting. After assessing the Maximum Sharpe ratio and minimum variance, the paper chooses the most outperformance stocks according to predicted data with the object of exhibiting the highest anticipated returns and lowest levels of risk. The second part employs the mean-variance optimization (MVO) technique and subsequently uses the asset weight approach to conduct portfolio evaluation. The methodology structure is demonstrated in Figure 2.

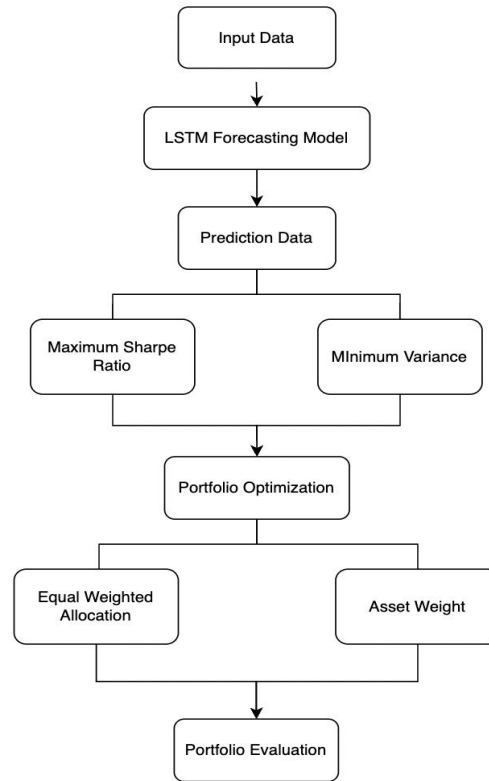


Figure 2: The overview of theoretical framework.

3.1. Prediction Model

The paper applies the LSTM forecasting model to predict stock prices and constructs a portfolio using the predicted results. The LSTM is a neural network structure. It can classify and process historical data to make predictions based on time series data [7]. The high effectiveness of reducing redundancy when processing historical data makes LSTM a well-spread prediction method for institutions and investors [8].

As an advanced version of RNN, LSTM possesses distinct benefits in effectively training data with long-term dependencies through the utilization of hidden layer units, commonly referred to as memory cells [9]. These memory cells have strong self-connecting functions that could help store network temporal state and incur three gates to control through: the input gate, the output gate, and the forget gate [10]. The previous two gates regulate the flow of the memory cell, where the input gate is utilized for inputting information into the cell state and the output gate is responsible for selecting the next hidden state [11]. The forget gate is used for finding out the information that should be dismissed from the cell [12]. Below is the structure of the LSTM network (See Figure 3):

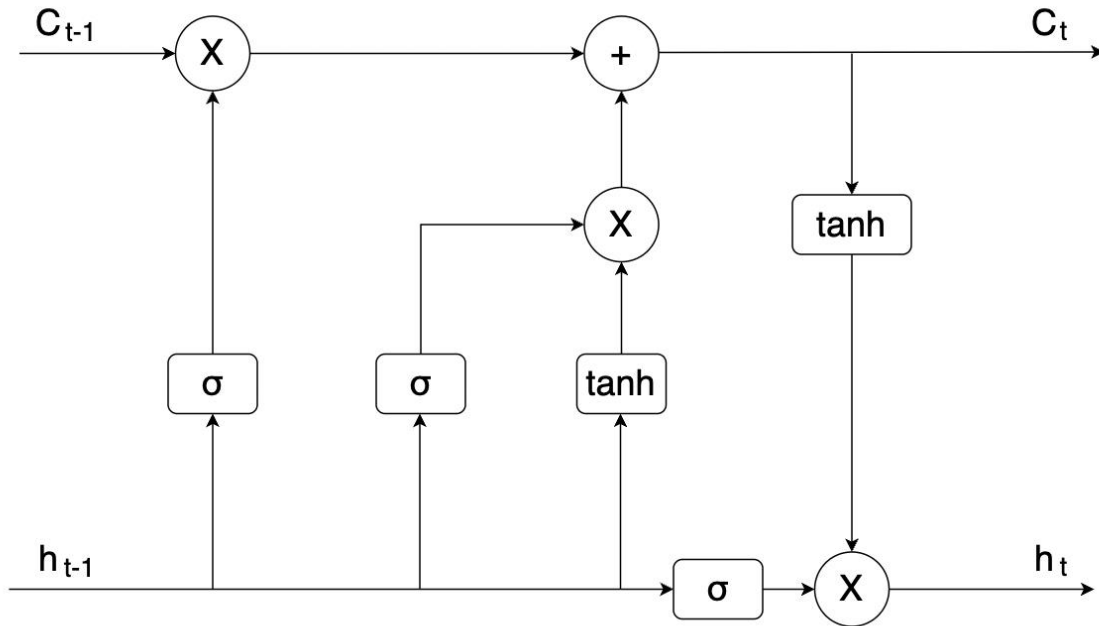


Figure 3: The overview of LSTM model.

The forget gate:

$$f_t = \sigma (W_f \cdot [h_{t-1}, x_t] + b_f) \quad (1)$$

The input gate:

$$i_t = \sigma (W_i \cdot [h_{t-1}, x_t] + b_i) \quad (2)$$

$$\tilde{C}_t = \tan h (W_C \cdot [h_{t-1}, x_t] + b_C) \quad (3)$$

Cell state:

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t \quad (4)$$

The output gate:

$$o_t = \sigma (W_o [h_{t-1}, x_t] + b_o) \quad (5)$$

$$h_t = o_t * \tan h (C_t) \quad (6)$$

3.2. Portfolio Optimization

The construction of portfolio is based on N different assets with different weights $w = (w_1, w_2, w_3, \dots, w_n)$ and total weight is equal to 1. Using mean-variance optimization (MVO) method, the paper calculates the expected return and variance of the portfolio with weights:

$$E(R) = \sum(S \times w) \quad (7)$$

In this formula, S represents each asset, and w represents the weight assigned to each asset in the portfolio.

$$Var(P) = \sum\sum(w_i \times w_j \times S_i \times S_j \times Cov(i, j)) \quad (8)$$

In this formula, Var(P) represents the portfolio variance, w_i and w_j are the weights of assets i and j, the S_i and S_j illustrates the signals for assets i and j, and Cov(i, j) refers to the covariance between assets i and j

4. Results

4.1. LSTM Forecasts

The paper collects 5-year historical data of selected stocks from Yahoo Finance during the period from 2018-01-01 to 2022-12-31 and sets an 80:20 train-test ratio for data prediction through the LSTM forecasting model. This study calculated the adjusted closing price and Mean Squared Error for each stock based on the specified criteria in the LSTM model. To be more specific, the LSTM model is configured with 50 units, and the range of epochs considered spans from 1 to 200. The results illustrate that the forecasted adjusted close price has better performance compared to the actual adjusted closing prices of the selected stocks over the five years (See Figure 4).

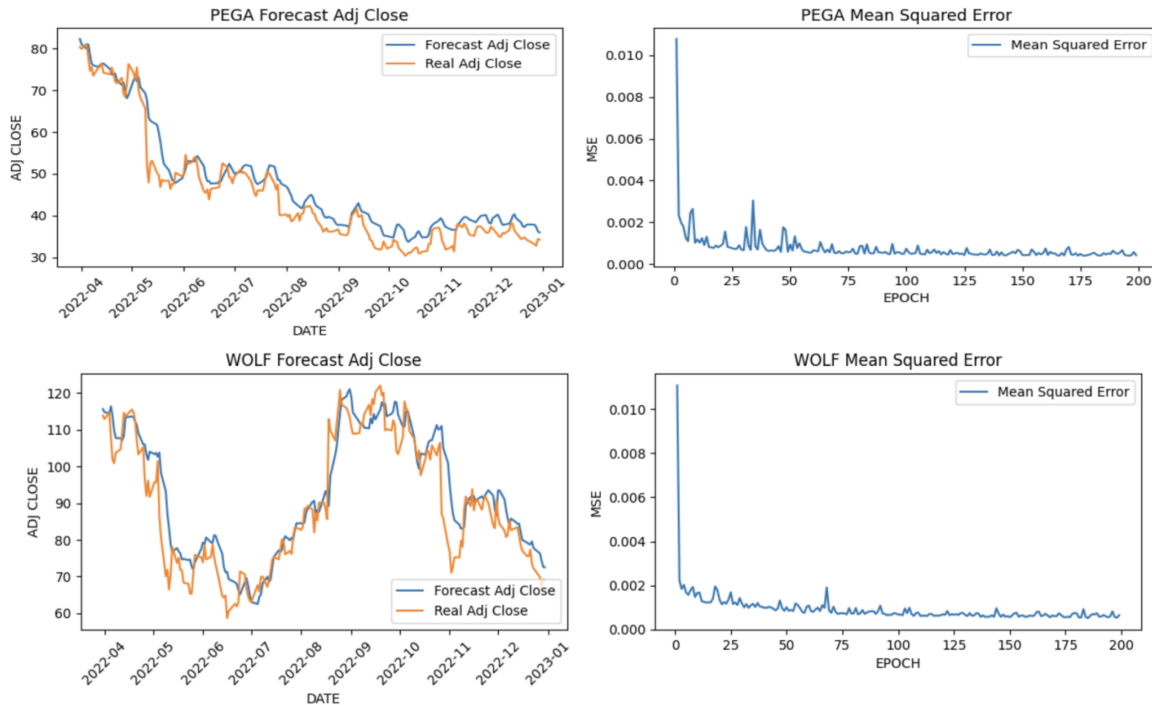




Figure 4: Predict values.

4.2. Portfolio Construction

With the intention of optimizing the portfolio, the paper uses the Mean-variance optimization method to calculate the portfolio's expected return, maximum Sharpe ratio and minimum risk. Furthermore, the study uses the Monte Carlo simulation model to generate simulations for more than 5000 portfolios in order to determine the appropriate weight of each stock within the created portfolio.

4.2.1. Monte Carlo Simulation Model

It is a popular method to repeat random sampling and do what-if analysis to reach the maximum Sharpe ratio in optimal portfolio. The simulation model is an efficient tool for evaluating multiple variables correlation and reviewing strategies. First, it generates portfolio weights based on random sampling and adjusts weights until the portfolio gets the highest Sharpe ratio. The Figure 5 below is efficient frontier, and the red star represents the maximum Sharpe ratio in the optimized portfolio [13]. And the related asset weights are shown in Table 3.

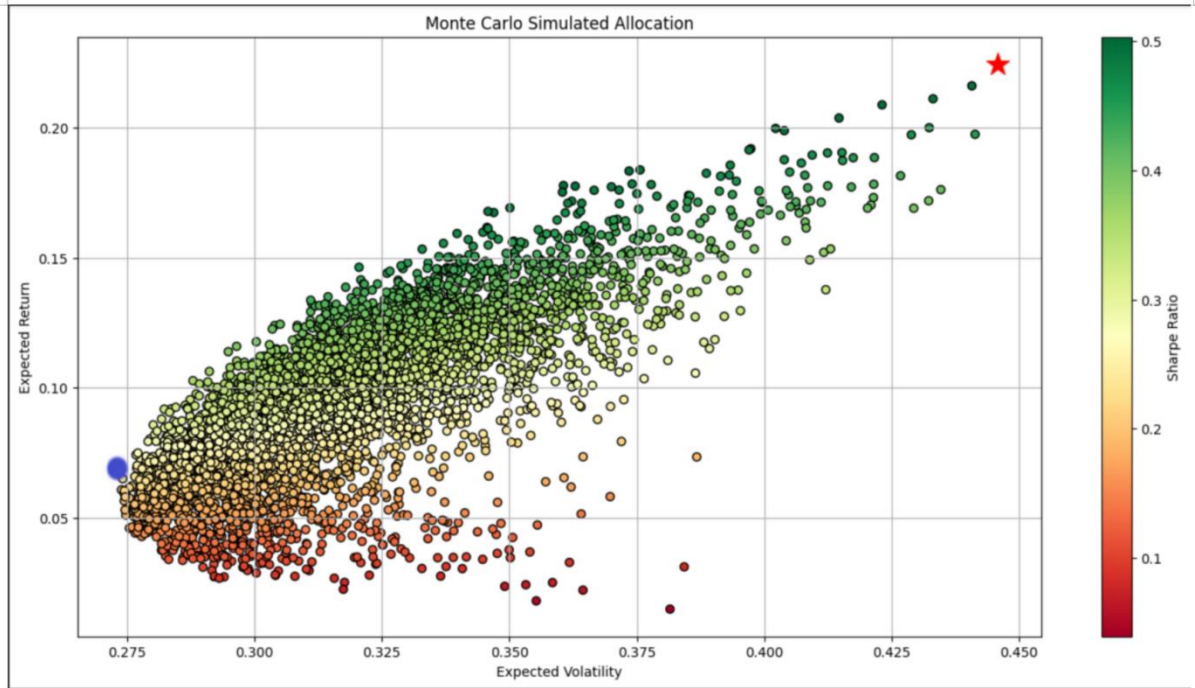


Figure 5: Efficient frontier.

Table 3: Asset weights.

Weights	Maximum Sharpe Ratio	Minimum Variance
PEGA	3.92	23.58
WOLF	0.21	19.0
SHEL	5.84	13.49
XOM	64.27	0.0
DOCN	25.76	43.93

4.2.2. Comparison with S&P500

To evaluate the portfolio's performance, the paper compares Maximum Sharpe Ratio and Minimum Variance of constructed portfolio with these data of S&P 500. The results are shown below in Figure 6:

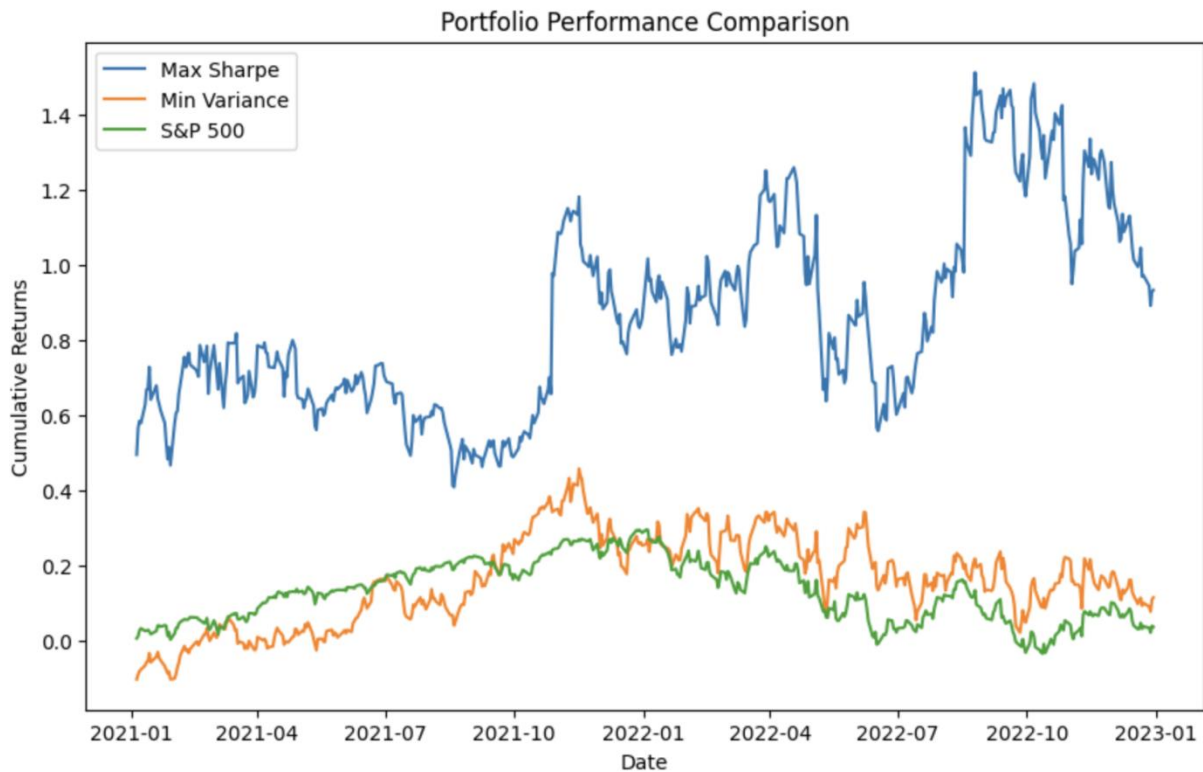


Figure 6: Comparison of with the S&P 500 index.

Obviously, the lines of Max Sharpe and Min Variance are better than the S&P 500 line. Hence, the constructed portfolio based on LSTM forecasting model has a better performance.

5. Conclusions

Predicting stock performance holds significant importance in the creation of investment portfolios. By using the LSTM model, the paper trains and tests a 5-year period dataset to get prediction data. In the context of portfolio evaluation, based on the predicted dataset, the paper uses the Minimum Variance Optimization method to approach the optimized portfolio with the maximum Sharpe ratio and minimum risk. Ultimately, this study undertakes a comparative analysis of the outcomes from the two models with S&P500 and concludes that the portfolio using the LSTM forecasting model has a strong performance.

There are several challenges in the process of portfolio construction. Firstly, historical data can not exactly forecast the future market. Secondly, the stock market experiences volatility as a result of a multitude of causes. Therefore, a good portfolio performance based on historical data does not guarantee profitability in the actual stock market.

Authors Contribution

All the authors contributed equally, and their names were listed in alphabetical order.

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