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Application of TTC and TTAS Mechanisms in Bed Allocation Problem in Chinese Universities

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Abstract: Dormitory beds are assigned to students at random in China each year. However, a big majority of students are dissatisfied with their results, which will have an impact on their academic performance. The goal of this paper is to help students choose a suitable bed by using an algorithm after the initial allocation. This research constructs a bed allocation model and firstly employ the top trading cycles algorithm, which fulfills individual rationality, Pareto-efficiency, and strategy-proofness. Notwithstanding, this paper finds that the chance of indifferences cannot be precluded, despite the fact that this article let students rank beds as indicated by their inclinations for rooms and beds. Subsequently, the top trading cycles algorithm is ineffectual for this issue. The top trading absorbing sets algorithm is then applied to the general domain to create individual rationally, Pareto-efficient, and strategy-proof matching. This paper finds that the top trading absorbing sets algorithm is a suitable solution for bed allocation problem in Chinese universities and suggests universities to apply this algorithm after randomly allocating beds to freshmen to enhance the satisfaction of students.

Keywords: top trading cycles algorithm, top trading absorbing sets algorithm, bed allocation

1. Introduction

Dormitory beds are assigned to students in China each year once they are admitted to universities. At the moment, most universities employ traditional methods to allocate dorms and beds, they allocate dormitories and beds based on students' grades or last names. However, it is normal for students to be dissatisfied with what they receive after moving into the dormitory. Some students, for example, prefer the first-floor room to the third-floor room, while others prefer the top bunk to the lower bunk.

It is critical that students live in a comfortable room with a comfortable bed. Since students will be living in the dormitory and sleeping on the same bed for at least four years, a comfortable living environment can have an impact on their academic achievement. That is, if a student who is not adept at climbing ladders is assigned to the top bunk, it is extremely likely that he or she will get wounded and will be absent from class, affecting his or her grades in some way. On the other hand, if this student is assigned to a lower bed based on his or her preferences, the number of situations mentioned above may reduce. The reason for this paper is to utilize a few mechanisms to further develop bed allocations after beds being haphazardly dispensed to freshmen in any case.

The bed allocation issue in universities is a highly controversial issue, various specialists have made papers to look at this issue. Contrary to conventional wisdom, the majority of researchers treat the room as a variable in their models and employ the top trading cycles algorithm to address the aforementioned issue. Regardless, the situation in China is exceptionally one of a kind comparable to that abroad. A large part of the time, students in China rest in aggregate rooms. Students have a variety of sleeping options because there are typically six people sharing a room and the rooms are very large. For example, light sleepers like the top bunk in the corner. That is the inspiration driving why the models referred to above are not sensible for China. In this paper, the model treats the bed in quarters as a variable, the issue of dorm distribution is refined into bed assignment which is essentially equivalent to "housing market", proposed by Shapley and Scarf in the study "On cores and indivisibility" [1]. In "housing market", there is a bunch of individuals, every one of them has strict inclination over a bunch of unified merchandise. At the end of the day, an individual cannot claim more than one good. Each individual is invested with a good and is permitted to exchange goods among themselves. Money-related compensation is prohibited throughout the entire interaction [1]. The conventional response for this kind of models is to apply an algorithm called the top trading cycle (TTC) algorithm.

The aforementioned algorithm and the "housing market" have been the subject of numerous papers. The TTC algorithm now clearly possesses variable properties. This algorithm is resistant to manipulation, and that implies that specialists can get a superior portion by uncovering their actual preferences in this algorithm [2]. Therefore, the aforementioned algorithm is the critical mechanism that ensures individual rationality, Pareto-efficiency, and strategy-proofness in the domain of absolute inclination [3]. Subsequently, this paper, without skipping a beat, uses the TTC algorithm to handle the bed assignment issue in college. Despite the fact that students in the model are approached to rank beds as indicated by their inclinations for rooms and beds, it is actually quite common for them to be indifferent between different beds. Under such circumstance, this exploration finds that the TTC algorithm may not be fitting for handling bed segment issue as the resulting matching is not actually Pareto-efficient.

Then, this paper endeavors to use another algorithm named the top trading absorbing sets (TTAS) algorithm, which obeys individual rationality, Pareto-efficiency and strategy-proofness on the overall space [4] to deal with bed assignment issue.

2. Mechanisms

2.1. The Bed Allocation Model

This paper assumes a bed allocation problem is a four-tuple $\{S, B, R, \omega\}$, where S represents a limited number of students and B represents a limited number of beds, both S and B should be in the same amount n ($|S| = |B| = n$), R refers to students' preferences for beds, and ω is an initial endowment matching. Then, let $\omega(i) = h_i$, where $i \in S$, $h_i \in B$, denote the original owners of beds. Subsequently, $\succ$ denotes that the student has strict preference over beds. For example, $b_2 \succ b_3$ means that student 1 strictly prefers b_2 to b_3. Let $\sim$ denote students' weak preferences over beds. That is, $b_4 \sim b_5$ describes student 1 is indifferent between b_4 and b_5. Likewise, $b_1 \succcurlyeq b_2$ means that student1 thinks b_1 is at least as good as b_2. Last but not least, in this model students are allowed to report indifferences.

An allocation (or coordinating) is a bijective guide μ: $S \rightarrow B$, where μ_i portray the bed distributed to student i under the assignment μ. A portion fulfills individual rationality assuming each student favors their last assignment μ_i to their original endowment ω_i. That is, for every student $i \in S$, $\mu_i \succ \omega_i$. Furthermore, this paper called an assignment is Pareto-efficient in the event that there doesn't exist other allocation λ with the end goal that, for all $i \in S$, $\lambda_i \succcurlyeq \mu_i$, and for some $j \in S$, $\lambda_j \succ \mu_j$. In this way, a component is Pareto-efficient in the event that it generally chooses a Pareto-efficient matching for each bed distribution issue. Thirdly, a strategy-proof allocation implies that each student cannot get

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a superior distribution by controlling their preferences, in different words, truth-telling is a rule procedure for all students.

2.2. Gale's TTC Algorithm

The bed designation issue is like house assignment issue somehow or another, and the old-style answer for this sort of issue is utilizing the TTC algorithm that is individual rationally, Pareto-efficient and strategy-proof on the space of strict preference [3].

Step 1: Let every student, right off the bat, point to their number one bed and each bed point to its underlying proprietor. There exists no less than one cycle and no cycles converge. Then, at that point, each bed and understudy in a cycle is eliminated.

Step n: Every student is allowed to point to their number one bed among the leftover ones, and each bed points to its underlying proprietor. There exists no less than one cycle and no cycles converge. Consequently, each bed and understudy in a cycle is eliminated.

Since everything is limited, the algorithm ceases when no students and beds left.

From the outset, this research assumes that all students could report strict preferences over beds after they thinking about both their preferences over rooms and their preferences over beds. Under such presumption, the TTC algorithm is a reasonable algorithm for bed designation issue in university. Subsequently, this paper applies a mechanism generalized the TTC algorithm as follows: First and foremost, this mechanism takes the inclinations of students reporting indifferences and transforms them into absolute sequences through (rigid or irregular) tie-breakers, and afterward applies the TTC algorithm [2].

Nonetheless, this paper observes that it is as yet workable for students to have differences despite the fact that they are approached to consolidate their preferences over rooms and their preferences over beds. On account of permitting students to report indifferences, the use of this system does not be guaranteed to prompt Pareto-efficient allocation. This will be shows in the accompanying example.

Example 1: Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$ be the group of students and beds, respectively. Let $\omega(s_i)=b_i$ for all $i \in \{1, 2, 3, 4, 5\}$ be the underlying endowment. The inclination profile is shown in Table 1 below.

Table 1: Inclination profile of five students.

s_1	s_2	s_3	s_4	s_5
b_2	b_3	b_4, b_5	b_1	b_2
b_1	b_1	b_3	b_5	b_4
b_4	b_2	b_2	b_4	b_5
b_3	b_5	b_1	b_3	b_1
b_5	b_4		b_2	b_3

It tends to be seen that there are two indifferences binary relation in Table 1, subsequent to utilizing arbitrary tie-breakers and applying the TTC algorithm, there are four potential consequences of the mechanism introduced above, in particular μ_1, μ_2, μ_3, μ_4.

$$\mu_1 = \{(s_1, b_1), (s_2, b_3), (s_3, b_5), (s_4, b_4), (s_5, b_2)\}$$
$$\mu_2 = \{(s_1, b_2), (s_2, b_3), (s_3, b_4), (s_4, b_1), (s_5, b_5)\}$$
$$\mu_3 = \{(s_1, b_1), (s_2, b_3), (s_3, b_4), (s_4, b_5), (s_5, b_2)\}$$
$$\mu_4 = \{(s_1, b_2), (s_2, b_3), (s_3, b_5), (s_4, b_1), (s_5, b_4)\}$$

It is easy to see that μ_1 and μ_2 is Pareto dominated by μ_3 and μ_4, respectively. Consequently, the TTC algorithm and the mechanism referenced above is not reasonable for taking care of bed assignment issue in university.

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2.3. Top Trading Absorbing Sets Algorithm

An absorbing set is a bunch of hubs A that fulfills two circumstances: (i) for any two hubs v_i, $v_j \in A$, there may be a way beginning with one then onto the next (inwards association), and (ii) there is absolutely no chance from any hub $v_i \in A$ to any hub $v_j \notin A$ (no inner-outside association). An absorbing set is paired-symmetric in the event that every one of its hubs has a place with a symmetric pair [4].

Two hubs v_i,$v_j \in V$ constitute a symmetric pair if there is a path from v_i to v_j and a path from v_j to vi [4].

Step 0: This algorithm chooses a need positioning of beds; that is, a finished, transitive and antisymmetric twofold connection over B.

Step 1-1: This algorithm lets every student pick their number one bed and each bed point to its underlying proprietor. There is no less than one absorbing set [5]. Then select the absorbing set in this step.

Step 1-2: This algorithm tracks down the matched symmetric-paired sets, if any. By assigning each student an underlying bed, the algorithm eliminates all students and beds that are in the paired-symmetric absorbing sets.

Step 1-3: This algorithm selects the leftover absorbing sets, if any. For every student choosing more than one bed, they are asked to select an exceptional bed to point utilizing the accompanying standard: the person point to the maximal bed with the most noteworthy need, not quite the same as their underlying bed.

Step 1-4: Then, at that point, in the subgraph framed by the groups and the bends chose in last step, there is generally somewhere around one cycle, and no two cycles converge. Subsequently, this algorithm assigns temporarily to every student in the aforementioned cycles the bed the person is selecting, but let them continue to participate in the algorithm.

Step n-1: Allow every excess student to choose their maximal beds from among the extra ones and every bed choose continuous owner.

Step n-2: This algorithm tracks down the matched symmetric-paired sets, if any. The algorithm eliminates all students and beds in the paired-symmetric absorbing sets by assigning each student an underlying bed.

Step n-3: This algorithm selects the leftover absorbing sets, if any. For every student selecting multiple bed, they are asked to select an exceptional bed to point utilizing the accompanying standard: From among the beds that have not yet been assigned to the individual in question, the one with the highest need points to the highest bed. This algorithm assumes all maximal beds have been designated to her essentially k times, subsequently, at that point, the person is asked to focus on the maximal bed with the most elevated need that has not been assigned to the person in question for k+1 times yet.

Step n-4: Then, at that point, in the subgraph framed by the groups and the bends chose in last step, there is generally something like one cycle and no two cycles cross. This algorithm assigns temporarily to every student in these cycles the bed the person is pointing, however let them stay in the algorithm.

Since everything is limited, the algorithm ceases when no students and beds left.

The following example shows the details of how the TTAS algorithm functions for a specific bed distribution issue.

Let $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}\}$ and $B = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}\}$ be the set of students and beds, respectively. Let $\omega(s_i) = b_i$ for all $i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the underlying endowment. The inclination profile is shown in Table 2.

Table 2: Inclination profile for ten students.

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
b_2	b_1	b_4, b_5	b_1	b_6	b_6, b_7	b_6	b_5, b_9	b_9, b_{10}	b_9, b_{10}
b_1	b_2	b_3	b_4	b_3		b_7	b_8		
			b_5	b_4					
				b_5					
s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}

The priority ranking is as follows: 1 > 2 > 3 > 4 > 5 > 6 > 7 > 8 > 9 > 10.
The detailed steps of how the TTAS algorithm works are shown below in Figure 1.

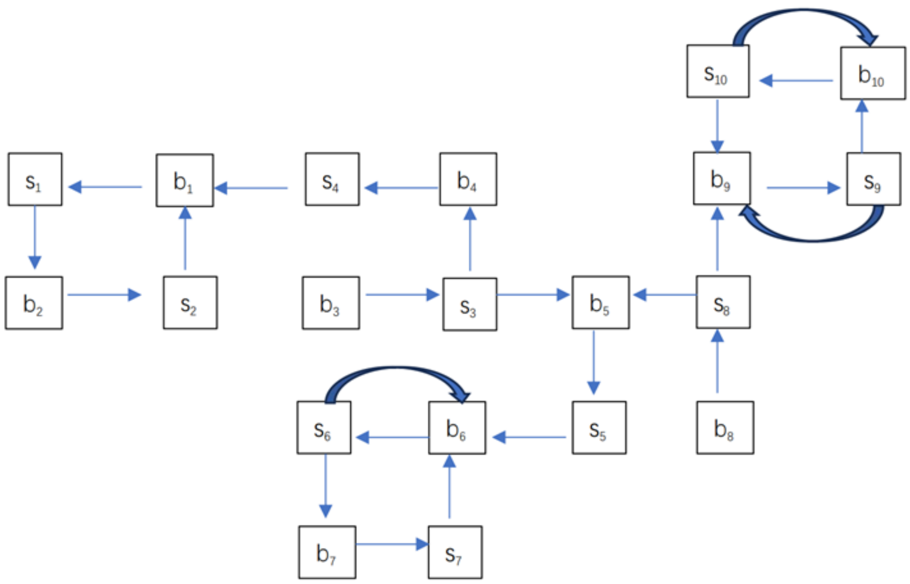

Figure 1: Step 1 of the TTAS algorithm.

It can be seen that there are three absorbing sets in Figure 1: $K_1 = \{s_9, b_9, s_{10}, b_{10}\}$, which is paired-symmetric. Thus, s_9 and s_{10} are removed by allocating b_9 to s_9 and b_{10} to s_{10}. Other absorbing sets are $K_2 = \{s_1, b_1, s_2, b_2\}$ and $K_3 = \{s_6, b_6, s_7, b_7\}$. For this situation, the need positioning over beds is applied, and the cycles $c_1 = (s_1, b_2, s_2, b_1)$ and $c_2 = (s_6, b_7, s_7, b_6)$ are composed. Then, the algorithm transitionally allocates b_2 to s_1, b_1 to s_2, b_7 to s_6 and b_6 to s_7.

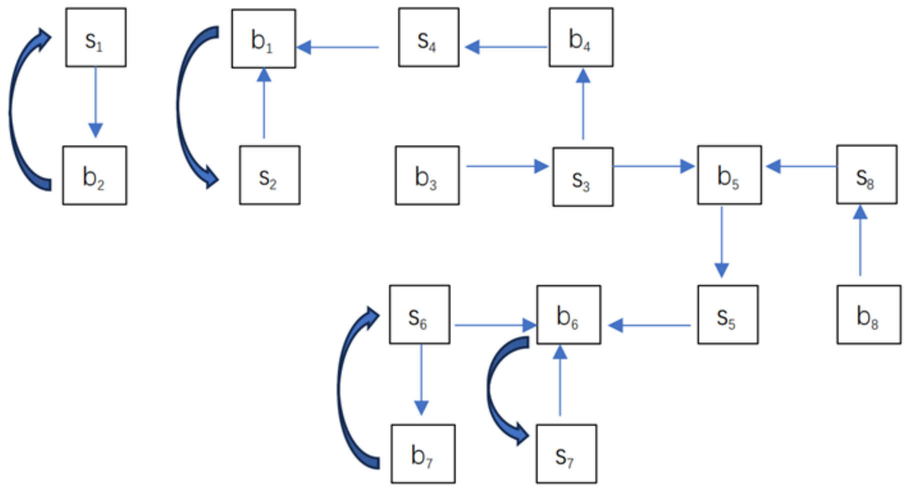

Figure 2: Step 2 of the TTAS algorithm.

It can be seen that there are three paired-symmetric absorbing set in Figure 2, $K_4 = \{s_1, b_2\}$, $K_5 = \{s_2, b_1\}$ and $K_6 = \{s_7, b_6\}$. These are then removed by allocating b_2 to s_1, b_1 to s_2, and b_6 to s_7.

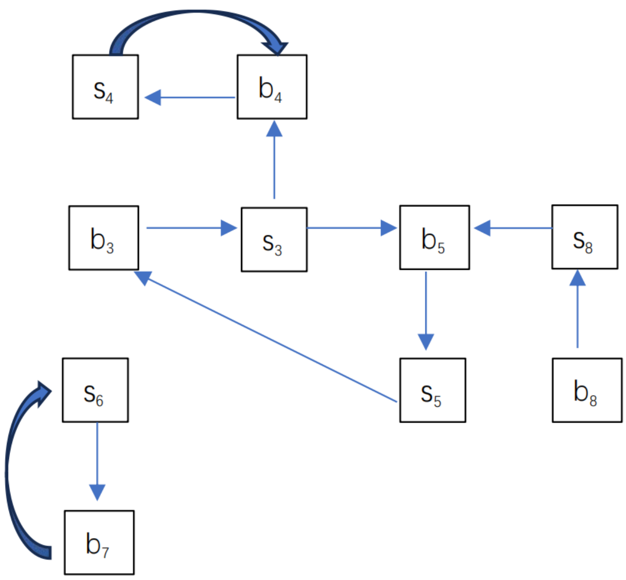

Figure 3: Step 3 of the TTAS algorithm.

Figure 3 shows two absorbing sets, $K_7 = \{s_6, b_7\}$ and $K_8 = \{s_4, b_4\}$, which are paired-symmetric. Hence, these are removed by allocating b_4 to s_4 and b_7 to s_6.

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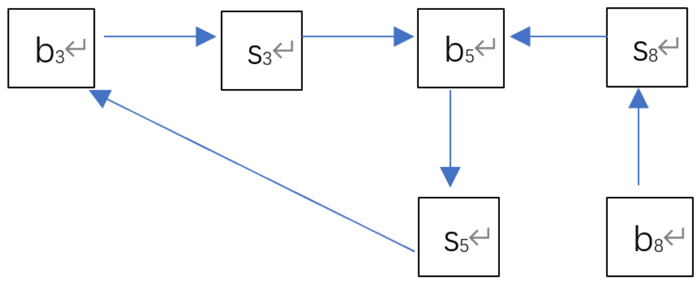

Figure 4: Step 4 of the TTAS algorithm.

There is only one absorbing set in Figure 4, which is $K_9 = \{s_3, b_3, s_5, b_5\}$. Then, the need positioning over beds is executed and thus forming a cycle $c_3 = (s_3, b_5, s_5, b_3)$. Subsequently, the algorithm provisionally allocates b_5 to s_3 and b_3 to s_5.

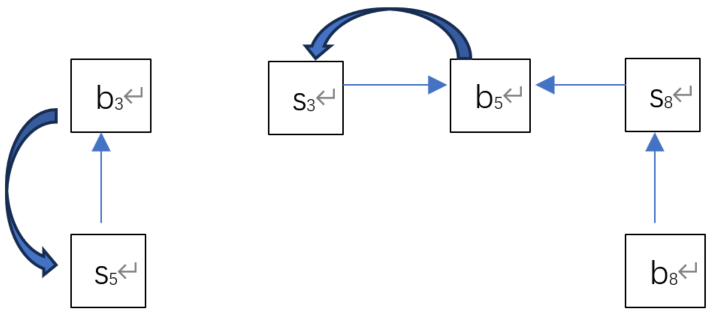

Figure 5: Step 5 of the TTAS algorithm.

There are two absorbing sets in Figure 5, $K_{10} = \{s_3, b_5\}$ and $K_{11} = \{s_5, b_3\}$, which are paired-symmetric. Thus, these are removed by allocating b_5 to s_3 and b_3 to s_5.

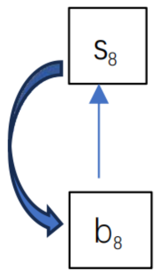

Figure 6: Step 6 of the TTAS algorithm.

It is clear that only s_8 and b_8 remain in this step, as shown in Figure 6, which form a paired-symmetric absorbing set $K_{12} = \{s_8, b_8\}$. Hence, this is removed by allocating b_8 to s_8.

Therefore, the resulting allocation is

$\mu = \{(s_1, b_2), (s_2, b_1), (s_3, b_5), (s_4, b_4), (s_5, b_3), (s_6, b_7), (s_7, b_6), (s_8, b_8), (s_9, b_9), (s_{10}, b_{10})\}$.

The TTAS algorithm decides an entitlement contingent upon the need positioning chose at Stage 0. A system ν is a TTAS mechanism assuming a boundary positioning exists to such an extent that the mechanism chooses, for each issue, the ascription chose by the TTAS algorithm with this need ranking [4]. For any need positioning, the system ν is individual rationally, Pareto-efficient and strategy-proof [4].

3. Discussion

A fundamental weakness of the TTAS algorithm is its high time intricacy, which emerges in light of the fact that the TTAS algorithm can exchange along some "terrible" cycles. That is, cycles where every one of the related individuals is as of now likely doled out to a bed in its favored set [6].

Alcalde-Unzu and Molis leave open whether or not the TTAS algorithm runs in polynomial time [4]. Aziz and de Keijzer answer this inquiry in the negative by showing a group of occasions on which the TTAS algorithm operates in extravagant time [7].

Nonetheless, flat mates likewise significantly affect students' ensuing scholastic presentation in various ways, which is a factor that this paper does not mention. Being roommates with higher-scoring fellows has positive influence on bachelors' accomplishment, especially for low-scoring fellows [8].

Strong proof indicates that peers in all actuality do matter for scholastic accomplishments, with flat mate explicit companion impacts being around 33% of the impacts of own capacity. These impacts are likewise observed to be non-direct, and to change across financial and geological backgrounds [9].

The students who pick their own flat mates in the class are better than the students who are haphazardly allocated by significant classifications as far as scholarly execution and complete execution in school, which demonstrates that the companion impact of flat mates on the development of college students assumes a significant part in scholastic presentation and thorough performance [10]. Acculturated quarters designation is helpful for lessening dorm inconsistencies, keeping away from struggle chances and guaranteeing grounds safety [11].

Thus, it is likewise essential to update bed distribution model by including students' preferences over flat mates for what's in store.

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4. Conclusion

Bed allocation problem is an important problem in university in China. The reason is that students will sleep on the bed they get for at least four years, quality rest can affect students' academic performance in some way. However, many students are not satisfied with their bed. This paper tries to use mechanisms to improve bed matching in university. By building a bed allocation model and applying the TTAS algorithm

The objective of this paper is to assist students with picking a satisfying bed by utilizing an algorithm after the underlying designation. In this exploration, this research develops a bed allocation model and first and foremost utilize the TTC algorithm. Notwithstanding, this paper finds that the chance of indifferences cannot be precluded despite the fact that this paper let students rank beds as per their preferences for rooms and beds. Subsequently, the TTC algorithm is ineffectual for this situation. The TTAS algorithm is then used to create individual rationally, Pareto-efficient, and strategy-proof matching on the overall space. As a result, this article finds that the aforementioned algorithm is a suitable solution for bed allocation problem in Chinese universities. Thus, this article suggests that Chinese universities could apply the aforementioned algorithm after randomly allocating beds to freshmen to enhance the satisfaction of students. However, flat mates likewise significantly affect students' ensuing scholastic presentation in various ways, which is a factor that this paper does not mention. The future study could focus on designing a new model by including students' preferences over flat mates for what is in store.

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