

Research on Optimal Portfolio Based on the Markowitz Model

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Abstract: This paper studies the Markowitz investment theory model. The study examined a portfolio of three companies - the Coca-Cola Company, McDonald's Corporation, and Apple Inc. This paper calculates the expected returns and the standard deviations of the Coca-Cola Company, McDonald's Corporation, and Apple Inc., respectively. In this paper, the expected return is used to measure the value of securities, and the standard deviation is used to measure the risk of securities, studying the development of three companies' stocks. At the same time, the relationship between the Coca-Cola Company, McDonald's Corporation, and Apple Inc. is discussed using covariance and correlation coefficient. Next, using the Markowitz investment theory model, one can draw the effective frontier of the investment portfolio and prove that all of the risk-return portfolios on the efficient frontier are the portfolios that can get the highest return with the fixed risk. Then, combining the efficient frontier and indifference curve, simulate a rational investor with a special risk appetite and a special expected return, verify the tangential point of the indifference curve, and this tangential point is the optimal proportional portfolio for this specific rational investor of these securities.

Keywords: Markowitz Model, Optimal Portfolio, Efficient Frontier, Expected Return, Risk

1. Introduction

Nowadays, people's living standards are constantly improving because of the development of society, and the per capita disposable income is also increasing. The data shows that the national per capita disposable income in 2014 was 20,167 yuan, and in 2022 was 36,883 yuan. As the country's per capita disposable income grows, more and more people begin to learn about investment. Securities investment refers to the investment process and the investment behavior of investors to earn bid-ask spreads, interest, and buying and selling stocks, bonds, funds, and other securities and their derivatives to gain capital.

Investment is all about running risks. "How to balance risk and return" has become the main issue investors pay attention to. The principle of investment is to minimize risk or maximize return. And everyone wants to obtain more benefits through investment. How to reduce the risk? Some investors will choose stocks with different trends to diversify the investment risk. So, how does this study know the optimal proportion of each stock in the portfolio? In order to solve this problem, Markowitz published an article in the Journal of Finance in 1952. The topic of this article was "Portfolio Selection - Effective Diversification of Investment", which marks the beginning of modern portfolio investment

theory. Markowitz's model is used to find the best investment portfolio and get the best benefits through historical data research.

Markowitz's asset allocation model provides a scientific theory for rational investors. The optimization analysis of asset portfolios helps rational investors get the maximum return under the condition of fixed risk.

According to modern portfolio theory, William Sharpe divided the stock risk into two parts: systemic and non-systemic risks. Systemic risk, in general, is the risk in the market of the specific industry; it can not be divided. And non-systemic risk usually means a specific risk. The impact of non-systemic risk on stocks is particularly large, but Markowitz found that it increases the number of stocks in the portfolio, and then the risks can be dispersed. "How a rational investor chooses the combination of the expected return and the risk in securities investment decision" is the central issue of the modern portfolio theory. It means a rational investor wants to get the maximum expected return at a fixed level of risk or minimizes the risk at a fixed expected return.

The mean-variance analysis and the portfolio-efficient frontier model are the most important parts of Markowitz's model. Markowitz's model uses the knowledge of probability theory and mathematical statistics in order to balance the return and risk of securities and optimize the asset portfolio. In this model, Markowitz uses expected return to measure the value of a security and variance to measure the risk. According to the covariance between asset portfolios, one can find the least-risk portfolio, which provides a scientific basis for rational investors when making decisions.

There is much research on using the Markowitz model for investment financial products. Xia Xue selected 4 securities in the Shanghai Stock Exchange Composite Index and verified the Markowitz portfolio model's role in investment [1]. Hens T and other authors looked into the differences between the variance of the return and the expected return on the same portfolio in the context of the Markowitz model [2]. Hens and other authors considered how much risk the rational investor can afford and analyzed the definition of an efficient portfolio and the optimization problems and underlying assumptions of the Markowitz model. Wang Xiaomin selected five stocks randomly and explained the application of the Markowitz model to minimize the risk of portfolio optimization [3].

2. Case Description

When rational investors make risky investments, they always choose several stocks to diversify their risk. Use several stocks' expected returns and variances to find the portfolio's efficient frontier.

This paper lists three companies- the Coca-Cola Company, McDonald's Corporation, and Apple Inc. -are selected as samples, and the data of each trading day from 12th May 2020 to 12th May 2021 of these 3 stocks are extracted for analysis, respectively [4].

The Coca-Cola Company is the largest beverage company in the world. It has a 48% share of the global market and two beverages of the world's top three drinks. The Coca-Cola Company ranked NO.64 on the 2017 Fortune American 500 list and NO.328 on the 2018 Fortune Global 500 list [5].

McDonald's Corporation is the largest fast-food conglomerate in the world. In 1984, the McDonald's Corporation sold about \$3.36 billion, and its assets were \$4.23 billion [6].

Apple Inc. is a technology company in America. As of June 2014, Apple Inc. has been the largest company by market capitalization all over the world for three years. Apple Inc. ranked NO.9 on the 2016 Fortune 500 list and No.6 on the 2021 Fortune Global 500 list. In 2021, Apple Inc.'s revenue reached \$365.8 billion [7].

In this paper, it is assumed that a rational investor invests in all three stocks. So, how can this study divide the three stocks to get the most benefit and the least risk?

3. Analysis on the problem

3.1. Use Excel to Analyze Stocks

First, calculate the expected return, variance, and standard deviation of the three company stocks according to the formulas. According to Markowitz's portfolio model, the bigger the standard deviation, the higher the investment risk of the stock and the more obvious volatility of investment returns, as shown in Tables 1 to Table 4.

Expected return on stock:

$$E(R_i) = \sum_{i=1}^{253} p_i r_i \quad (1)$$

Risk on stock:

$$E(R_i) = \sum_{i=1}^{253} p_i r_i \quad (2)$$

Table 1: Expected, Var, and StDev of 3 stocks

| | Expected return | Var | StDev |
|------------|-----------------|---------|---------|
| Coca-Cola | 0.0968% | 0.0187% | 1.3675% |
| McDonald's | 0.1180% | 0.0136% | 1.1678% |
| Apple | 0.2080% | 0.0491% | 2.2168% |

Then, calculate the correlation coefficient of the three stocks. The correlation coefficient can represent the relationship between every two stocks in the same portfolio. When the numeric value is more than 0, it represents a positive relationship between two stocks; otherwise, the relationship is negative.

Table 2: Correlations of 3 stocks

| Correlations | Coca-Cola | McDonald | Apple |
|--------------|-----------|----------|--------|
| Coca-Cola | 1 | 0.5576 | 0.2155 |
| McDonald's | 0.5576 | 1 | 0.2923 |
| Apple | 0.2155 | 0.2923 | 1 |

Use Excel to calculate the covariance between the three stocks. When the covariance is higher, the correlation between the two stocks is stronger.

When the covariance is positive, that represents the two stocks will change in the same direction; conversely, the two stocks will change in opposite directions; when the covariance is 0, there is no correlation between the two stocks.

Table 3: Covariance of 3 stocks

| Covariance | Coca-Cola | McDonald | Apple |
|------------|-----------|----------|---------|
| Coca-Cola | 0.0186% | 0.0089% | 0.0065% |
| McDonald's | 0.0089% | 0.0136% | 0.0075% |
| Apple | 0.0065% | 0.0075% | 0.0489% |

Use Excel to calculate the beta coefficient. The beta coefficient is a financial instrument. It can be used to assess security systemic risk in order to measure the degree of fluctuation of a security or a portfolio in the whole market. And the higher the absolute value of the beta coefficient, the higher the degree of fluctuation of the stock's return relative to the whole market; on the contrary, the change range is smaller. The rational investor can learn about the future market risk situation that the security

will face through the beta coefficient. It helps rational investors understand the investment market trends and reduces investment risks.

Table 4: Beta coefficient

| | Beta coefficient |
|------------|------------------|
| Coca-Cola | 0.7641 |
| McDonald's | 0.6206 |
| Apple | 1.3332 |

3.2. Analysis of the portfolio

According to the data in Table 1, Figure 1 can be obtained.

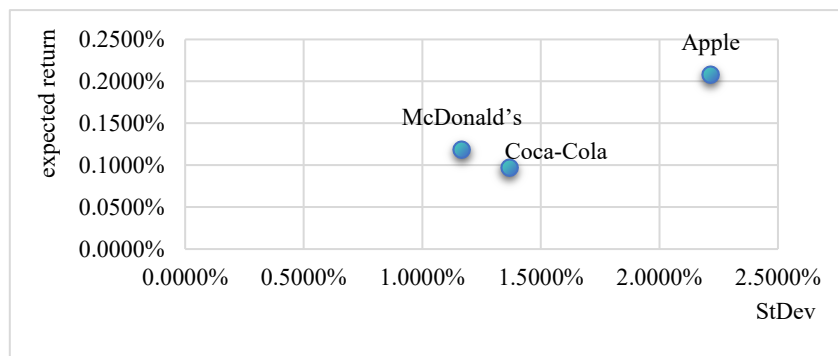


Figure 1: Expected return and StDev (Photo credit: Origin)

The horizontal coordinate in Figure 1 is the standard deviation, representing the risk. And the ordinate is the expected return. The Coca-Cola company's expected return and standard deviation are 0.0968% and 1.3675%, respectively. McDonald's Corporation's expected return and standard deviation are 0.1180% and 1.1678%, respectively. Apple Inc.'s expected return and standard deviation are 0.1180% and 1.1678%, respectively.

Next, use Excel to generate 20 sets of random ratios and calculate each portfolio's weighted expected return and weighted standard deviation. Through these 20 sets of data, the effective boundary of the three stock portfolios can be drawn.

Rational investors usually hate risk and prefer return. When the risk level is fixed, the portfolio with the highest return will be selected; when the expected return is fixed, the portfolio with the least risk will be selected. A portfolio that has all the above conditions is an effective frontier. The efficient boundary is the set of these portfolios.

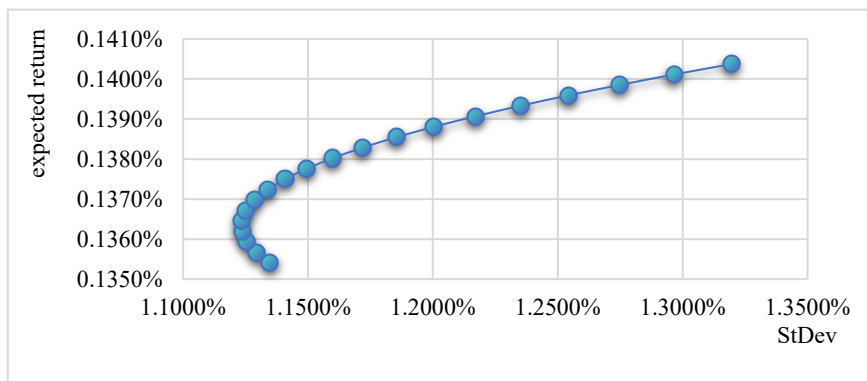


Figure 2: Efficient boundary (Photo credit: Origin)

As shown in Figure 2, the horizontal coordinate represents the risk, and the vertical coordinate represents the return. And the points on this curve represent the feasible portfolios of securities. Obviously, the points on the curve in Figure 2 represent the portfolio securities with the highest return for the same risk.

4. Suggestion

4.1. Efficient Frontier

The efficient frontier represents the investment portfolio's relationship between the risk and the expected return. It is drawn as a curve with the risk as the horizontal axis and the expected return as the vertical axis. All the portfolios on this curve can obtain the maximum return with the fixed risk [8-10].

The efficient frontier is the top half part edge of the feasible region of a portfolio. The efficient frontier must be convex. And why can only the upper edge be called an efficient frontier? Although the portfolios on the right side and those on the efficient frontier have the same degree of risk, the portfolios on the right side have a lower return and are inefficient. The portfolios on the left side of the efficient frontier and those on the efficient frontier have the same degree of expected return, but the left side has lower risk; it is an impossible event!

The efficient frontier is the most important and useful thing for establishing optimal portfolio and security valuation. The job of a manager to manage the fund will usually manage large assets, and it is essentially how to place the risk-return value of their portfolios on the efficient frontier. The portfolios around the efficient frontier can use a diversified investment method to improve the return rate without improving the risk or reduce the risk without reducing the return rate.

4.2. Select Optimal Portfolio on Efficient Frontier

4.2.1. Indifference Curve

The indifference curve means that the combination of securities at each point on the curve is different, but the investor gets the same degree of utility from every point. If the consumer chooses no matter which points on the curve, they will satisfy him without difference at all. All of the points are equally desirable for him.

The indifference curve is a curve obtained according to the aversion of a specific investor's expected rate and risk and the requirement of risk compensation according to the expected rate.

When on the same coordinate plane, there is an infinite number of indifference curves between every two indifference curves. Based on the origin on the coordinate axis, If the indifference curve is far from the origin, it illustrates that the indifference curve has a higher utility level; if the indifference curve is close to the origin, it illustrates that the indifference curve has a lower utility level. The indifference curves are convex toward the origin. And every two indifference curves will never intersect on the same coordinate.

4.2.2. Optimal Portfolio

Suppose that an investor with a specific risk appetite and expected rate of return would like to invest in the Coca-Cola Company, McDonald's Corporation, and Apple Inc. portfolio. Draw the indifference curves according to the risk coefficient and other indicators of the investor. And it is obvious that the tangent point is the optimal portfolio with the expected return and the risk.

5. Conclusion

5.1. Markowitz Modern Portfolio Theory

In 1990, Markowitz won the Nobel Memorial Prize in Economic Science because of his theory of portfolio selection. He developed an operable theory of a selective portfolio of assets that is rigorously stated under the uncertainty conditions - the mean-variance methodology.

In Markowitz's modern portfolio theory, changing a portfolio price is treated as a random variable. Use the mean to measure the return and the variance to measure the risk. Next, consider each security ratio in the portfolio as a variable, and it can change the main portfolio problem to a quadratic programming problem under linear constraints. Finally, make the decisions about investing according to the investor's preferences.

Markowitz thinks that the risk of a portfolio is not only related to the individual risk of the individual stocks in the portfolio but is also affected by the relationship between the various stocks (covariance). In the process of securities investment, it is important to carefully analyze the value and risk of each stock and pay more attention to the correlation between the several stocks when investing. Although each stock in the same portfolio carries a certain risk, as long as the stocks in the portfolio are not perfectly related, the portfolio can spread the risk when investing. The lower the correlation between the stocks in the portfolio, the better the effect of spreading non-systemic risk.

Markowitz's paper marks the beginning of the modern portfolio theory. The theory about the problem of portfolio selection under uncertain conditions has gradually developed into the foundation of contemporary financial investment theory. The Sharpe Single Index Model, the Capital Asset Pricing Model, and the Arbitrage Pricing Theory derived from the Markowitz Modern Portfolio Theory, with the Efficient Markets Hypothesis and the Behavioral Finance Theory, constitute modern portfolio theory. Because of that, Markowitz shared the Nobel Prize in Economic Sciences in 1990 with Merton Miller and William Sharp.

5.2. Limitation of the Markowitz Modern Portfolio Theory

In Markowitz's investment theory model, investors are assumed to be rational investors. This assumption does not correspond to reality because, as a people, they must have irrational behaviors. Real-life investors have certain cognitive biases and can not be completely rational.

Investors' attitudes to risk are not fixed. In Markowitz's investment theory model, it is assumed that the investors are all risk-averse. But in real life, the two characteristics of risk aversion and risk preference can be reflected in an investor together.

In the Markowitz investment theory model, the standard deviation represents the stock's risk. However, it only works under some specific conditions. So there are also some problems. The standard deviation is used to measure the fluctuation of the price originally, including the fall and rise degree of the price. But risk really means that a price fall causes investors to lose money. So, only using the standard deviation to measure the investment risk is not really accurate.

In fact, the most fundamental flaw with Markowitz's investment theory model is the amount of historical data required when there is a large number of securities. And the cost of redistribution is very high.

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