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Improved Hedge Fund Portfolio Optimization Using 2-Step Covariance Matrix and Fund Transaction Costs

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Abstract: This study focuses on fund portfolio investments in the Chinese market. The application of classic portfolio optimization methods encounters several issues when applied to fund portfolios. For example, issues such as the non-normal distribution of returns on funds or fund portfolios, turnover rate limitations in fund investments, and liquidity constraints of fund assets, which can lead to transaction costs and opportunity costs, are prevalent challenges. The existence of these issues can compromise the effectiveness of classic portfolio optimization methods like Mean-Variance Optimization. This may result in a reduction of accuracy in determining the portfolio's optimal weights, a deviation of actual trading results from the model's optimal expectation, and may even render the optimal weights impractical in real-world scenarios. To address these challenges, this paper integrates the 2-Step covariance matrix method (2-Step method) and the measurement of fund transaction costs into the portfolio optimization process. The paper finds that the 2-Step method, compared to the baseline, can indeed improve the risk-return indicators of the potimal fund portfolio. The inclusion of the transaction cost can effectively control the turnover frequency of the portfolio. Even after accounting for these costs, the 2-Step method continues to exhibit a significant improvement effect compared to the baseline.

Keywords: Portfolio Optimization, Covariance Matrix, Hedge Fund, Transaction Cost, Fund of Fund

1. Introduction

In recent years, hedge funds have flourished in the Chinese market, and a multitude of high-quality hedge fund assets have emerged, earning favor from investors. Consequently, methods to effectively combine superior hedge fund assets to form portfolios that balance returns and risks have gradually captured investors' attention. However, the application of classic portfolio optimization methods in hedge fund portfolios encounters several issues:

(1) The distribution of hedge fund returns deviates from the normal and t-distributions, for instance exhibiting significant kurtosis or skewness, distinct from traditional stock returns. This leads to inaccurate risk estimations, neglect of tail risks, unstable optimization results, and potentially overly concentrated positions.

(2) In actual hedge funds transactions, various factors such as portfolio turnover limitations, fund liquidity constraints, and the trading complexity relative to other secondary market tradables bring

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about significant trading costs. Traditional frameworks rarely incorporate quantitative penalties for fund portfolio position adjustments, which may result in an inability to actualize the model's optimal portfolio or significant deviations from the optimal results in practical operations.

This paper quantitatively integrates the 2-Step method, aiming to improve hedge fund risk estimation by enhancing covariance matrix estimation methods, and fund transaction costs into the optimizer. This effort seeks to create a more effective fund portfolio with improved return-risk ratios and practicality.

2. Literature Review

First and foremost, when it comes to the realm of portfolio optimization, Mean-Variance Optimization (MVO) plays a foundational role. A vast amount of academic research has been conducted around MVO, leading to numerous enhancements and adaptations. In the industry, MVO is widely regarded as a fundamental starting point for various investment strategies. However, MVO has been widely criticized over the years for various reasons, a primary concern being the sensitivity of optimal results to model inputs [1-2]. Such sensitivity hinders its adaptability to short-term market regime-switching, and makes it challenging to achieve exceptional out-of-sample performance [3]. The academic community has consequently proposed various extensions to MVO, new optimization objective functions, and new asset allocation methods, such as Max Diversification Ratio [4], Risk Parity [5], Mean-CVaR [6], Mean-semivariance [7], and Efficient CDaR [8], each making breakthroughs in various dimensions. In this paper, for more robust evaluation and control of research variables, the globally accepted "most robust" MVO framework that excludes the impact of expected returns, the Global Minimum Variance (GMV) portfolio [9], is chosen as the starting point of the discussion.

Another way to mitigating the shortcomings of MVO is enhancing the precision of returns and risk estimations, primarily by refining methods used for estimating the covariance matrix to augment its robustness and eliminate noise. According to Peter J. and his collegues, their research introduced the Minimum Covariance Determinant estimator, a robust estimator that considers the model's stability when facing outliers [10], and Rousseeuw, P.J et al. further proposed the Elliptic Envelope method for anomaly detection [11]. Besides, other studies presented several shrinkage estimators, combining the sample covariance matrix with a structured target [12-14]. Estrada, J. explored a risk measurement method focusing on adverse price fluctuations - semivariance [15]. These methodologies have been proven to significantly bolster the MVO framework. It's also worth noting that improving the accuracy of expected return estimations is another viable direction, with numerous well-constructed methodologies present in both academia and industry, details of which will not be further delved into within this paper.

The 2-Step method, pioneered by the study conducted by Arnott, R.D. et al. [16], refined EWMA by applying a 2-Step volatility management approach that has demonstrated efficacy in increasing risk-adjusted returns in Factor Investing. This paper further extends the 2-Step concept to more risk measurement methods. Considering the characteristics of hedge fund products and the inherent cyclicity and seasonality in the Chinese market, this paper adjusted the hyperparameters of the model to explore whether the combination of these methods can solve the current difficulties faced in fund investment.

Lastly, the return distribution of funds or fund portfolios (such as discretionary stock/CTA strategies) exhibiting sharp peaks and fat tails is largely influenced by the time series variability in volatility levels. This observation can be traced back to research such as the research [17], which mentions the uneven distribution of risk events over time. There sharp peaks and fat tails are also due to characteristics of specific investment strategies, like the inherent leverage in CTA strategies. Generally, short-term volatility persists more reliably than short-term return predictions, making it a

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good predictor of future risk. To highlight short-term fluctuations, commonly, EWMA is used to prioritize recent data, tuning the model for short-term variability. The 2-Step method we implemented pursues a similar goal, balancing the model between long-term and short-term fluctuations, and adapting to the time-varying volatility of multiple funds, making the model more adaptable to short-term market regime-switching.

3. Data and Methodology

3.1. Data

The fund data used in this article comes from Go-Goal, a reputable hedge fund data platform in China. Various fund indices, including long-only equity, long-short equity, equity index-enhancement, CTA, macro strategy, and multi-strategy, were chosen as inputs for the portfolio optimization model. Go-Goal's fund universe mainly includes non-institutionalized 'sunshine' hedge fund products with open-to-public performance, issued domestically through trust platforms, brokers, fund subsidiaries, and hedge funds' own platforms. Indices are calculated using a weighted average method, considering dividend reinvestment. More details are available at (https://cloud.12345fund.com/home). Due to the nascent development of transparent hedge funds in the China market, this study utilizes hedge fund data available since December 31, 2015, collected on a weekly frequency.

3.2. Risk Measurements and the 2-Step Covariance Matrix

For risk measurement methods, this paper employs various approaches including the sample covariance matrix (Cov), exponentially weighted covariance matrix (Exp_Cov), semi-covariance (Semi_Cov), Ledoit-Wolf Shrinkage (LW_Const_Corr, LW_Single_Factor, LW_Regularized), Minimum Covariance Determinant estimator (Min_Cov_Det), Elliptic Envelope Method (Elliptic_Envelope), and Oracle Approximating Shrinkage Estimator (OAS). Based on these measurements, a further 2-Step transformation is performed. The calculation method is as follows:

(1) Set a baseline model: Cov、Exp_Cov、Semi_Cov、LW_Const_Corr、LW_Single_Factor、LW_Regularized、Min_Cov_Det、Elliptic_Envelope, OAS.
(2) Calculate the long-term risk matrix $\Sigma_{baseline}$ corresponding to different baseline models. The rolling window is adaptable, ranging from a minimum of one year to a maximum of three and a half years, based on a rule of thumb derived from the characteristics of the beta rotation cycle in China's market in recent years.
(3) Calculate the average risk matrix $\Sigma_{average}$ by averaging K short-term risk matrices.
(4) Obtain the 2-Step covariance matrix.

$$\Sigma_{2-Step_i} = \begin{cases} \Sigma_{baseline_i}, & \Sigma_{baseline_i} \in [(1-\alpha) * \Sigma_{average_i}, (1+\alpha) * \Sigma_{average_i}] \\ \Sigma_{average_i}, & \text{Others} \end{cases}$$

In this paper, α is set to 0.05, and rolling window K is set to 8, 16, and 24 weeks.

To intuitively understand this 2-Step covariance matrix, when the long-term volatility is close to the short-term volatility, we believe that the volatility environment of all assets (in the past six months) has not undergone regime-switching, so the risk matrix continues to use $\Sigma_{baseline}$. However, when the long-term volatility exceeds the range of short-term volatility, it is believed that the volatility condition of some or all assets has changed, and therefore it is necessary to use the short-term average risk matrix $\Sigma_{average}$ to replace $\Sigma_{baseline}$. Since our model inputs are hedge fund indices, which inherently exhibit more robust and less volatile performances compared to individual funds or small fund portfolios, we have chosen to apply a relatively strict volatility interval constraint parameter α.

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3.3. Integrating Fund Transaction Costs into the Objective Function

As mentioned earlier, Minimum Variance (Min-Var) is used as the starting point for our research. We first include the transaction cost term into the Min-Var objective function:

$$Minimize \qquad \lambda * w^T \Sigma w + c * \Sigma |w - w_{pre}|$$

Where,Σ represents the covariance matrix,the constant term c represents transaction costs, the constant term λ is a ‘balancing’ coefficient,w_{pre} is the weight of the portfolio in the previous period, and w is the portfolio weight to be optimized.

However, the introduction of transaction cost calculation in this objective function makes the original problem challenging to solve. A method is employed to address this issue, which transforms the problem into two continuous convex optimization problems. The steps are as follows:

Step 1: Solve the Min-Var problem without the transaction cost term to get optimal weights w^*.

Step 2: Define cost coefficient arrayc^*

$$c_i^* = \begin{cases} c_i, & w_i^* \geq w_{pre\_i} \\ -c_i, & w_i^* < w_{pre\_i} \end{cases}$$

Step 3: Obtain optimal weights $w_{after\_cost}^*$:

$$Minimize \qquad \lambda * w^T \Sigma w + c^T w - c^T w_{pre}$$

$$s.t. \quad \begin{cases} 0 \leq w_i \leq w_{pre\_i}, & c_i^* < 0 \\ w_{pre\_i} \leq w_i \leq 1, & c_i^* \geq 0 \end{cases}$$

3.4. Transaction Costs and Coefficient Lambda

For secondary market assets (most representatively, such as stocks and futures), the transaction cost c is easy to determine. However, there is no fixed standard for fund transactions. This paper estimates the trading friction of funds from the perspective of opportunity cost. Suppose fund transactions, such as redemptions, result in a capital ‘lock-up’ period of T days, and considering 250 trading days in a year, with an annualized capital opportunity cost of R, the estimated transaction friction is:

$$c = \frac{T}{250} * R$$

The opportunity cost can refer to various aspects, for example, the risk-free rate, the yield of monetary funds, and the cost of capital etc.

The coefficient λ in the objective function acts to balance the risk term with the cost term, similar to the risk aversion coefficient in MVO. When transaction costs are high, λ allows the role of risk not to be entirely dominated by high costs, preventing the portfolio from being constrained by trading friction and becoming stagnant.

$$\lambda = Max(\frac{c}{w_{pre}^T \Sigma w_{pre}}, 1)$$

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3.5. Extension from Min-Var to Max-Sharpe

Besides examining the Min-Var portfolios, we also tried to take expected returns into account. For simplicity, we avoided using complicated return prediction models, instead assuming that funds' historical returns could represent future performances. Max-Sharpe is a classic optimizer that considers both returns and risk. The study also transformed it into a simplified convex optimization problem [18]. Under certain assumptions (such as the sum of weights being equal to one and the existence of a feasible domain where the portfolio's expected return rate is higher than the risk-free rate), its equivalent quadratic programming problem is as follows:

Obtain optimal weights w^*:

$$Min\ y^T \Sigma y$$

$$s.t. \left(\mu - r_f e\right)^T y = 1$$

$$(y, \kappa) \in \chi^+$$

$$\chi^+ := \left\{ w \in R, k \in R \middle| \kappa > 0, \frac{w}{\kappa} \in \chi \right\} \cup \{0,0\}$$

$$w^* = \frac{y}{\kappa}$$

Further considering the transaction cost term within the Max-Sharpe framework:

Step 1: Solve the Max-Sharpe Problem without costs to obtain optimal weights w^*:

Step 2: Define cost vector c^*

$$c_i^* = \begin{cases} c_i, & w_i^* \geq w_{pre\_i} \\ -c_i, & w_i^* < w_{pre\_i} \end{cases}$$

Step 3: Obtain final optimal weights $w_{after\_cost}^*$:

$$Minimize \qquad \lambda * y^T \Sigma y + c^{\mathrm{T}} y - c^T (w_{pre} * \kappa)$$

$$s.t. \quad \begin{cases} 0 \leq y_i \leq w_{pre\_i} * \kappa, & c_i^* < 0 \\ w_{pre\_i} * \kappa \leq y_i \leq \kappa, & c_i^* \geq 0 \end{cases}$$

$$s.t. \left(\mu - r_f e\right)^T y = 1$$

$$(y, \kappa) \in \chi^+$$

$$\chi^+ := \left\{ w \in R, k \in R \middle| \kappa > 0, \frac{w}{\kappa} \in \chi \right\} \cup \{0,0\}$$

$$w_{after\_cost}^* = \frac{y}{\kappa}$$

4. Analysis and Discussion

4.1. Empirical Data and Results

This section primarily displays empirical results through historical back-testing. The settings are as follows: The back-testing period is from January 2017 to September 2023, with all data from 2016 retained for initializing the covariance matrix. Portfolios are monthly-rebalanced, and the optimization objectives include Min-Var and Max-Sharpe. Taking into account the scarcity of high-quality hedge fund products, inherent strategic cyclicality, and the necessity for risk management, asset diversification is emphasized as a key in real-world investments. Therefore, in the optimization, a constraint has been implemented to ensure that a single asset does not exceed a 60% allocation.

4.1.1. Empirical Results of the 2-Step Covariance Method under Min-Var Optimizer

To begin with the Min-Var Optimizer, Table 1 displays the return-to-risk ratios, Annualized Return (Ret), Annualized Volatility (Std), Annualized Downside Volatility (DStd), Sharpe Ratio (SR), Sortino Ratio (Sortino), and Maximum Drawdown (MDD), of optimal portfolios corresponding to 2-Step and baseline models, excluding the fund transaction cost item c and the coefficient λ. Table 2 presents the return-to-risk ratios of 6 selected hedge fund indices for the same period.

From Table 1, it's evident that across the entire period, compared to the baseline, all 2-Step methods manage to reduce the portfolio volatility to varying degrees. Moreover, nearly all 2-Step methods can significantly enhance the portfolio's Sharpe Ratio or at least match the baseline. Although Table 2 reveals a dominant performance of CTA within the interval, primarily due to the CTA bullish market condition between 2020 and 2021, most of the 2-Step optimized portfolios, even with a maximum of only 60% CTA holdings, could achieve a lower annualized volatility compared to holding CTA only. This highlights the significance of diversification.

Table 1: Comparison of risk-return metrics between 2-Step and baseline methods without the transaction cost c.

	SR(2-Step)	SR	Std(2-Step)	Std
Cov	1.53	1.51	3.54%	3.63%
Elliptic_Envelope	1.55	1.41	3.63%	3.69%
Exp_Cov	1.53	1.52	3.53%	3.59%
LW_Const_Corr	1.51	1.50	3.55%	3.64%
LW_Regularized	1.49	1.50	3.69%	3.69%
LW_Single_Factor	1.49	1.50	3.57%	3.63%
Min_Cov_Det	1.56	1.36	3.63%	3.70%
OAS	1.48	1.50	3.68%	3.68%
Semi_Cov	1.50	1.46	3.56%	3.64%
Equal_Vol		0.94		5.09%
Equal_Weight		0.78		6.02%

Table 2: Risk-return metrics of 6 selected hedge fund indices.

	Ret	Std	SR	MDD
CTA	7.71%	3.77%	2.046	3.18%
Equity Index-Enhancement	6.69%	12.82%	0.522	22.06%
Equity Long-Only	3.07%	8.73%	0.351	14.13%
Equity Long-Short	4.44%	6.99%	0.635	12.70%
Macro	3.79%	6.35%	0.596	14.53%
Multi-Strategy	4.18%	5.52%	0.758	8.89%

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4.1.2. Empirical Results of Introducing Transaction Cost and Coefficient Lambda

Based on Table 1, 2-Step method has been primarily proved to be effective in fund portfolio optimization. Consequently, we introduce the cost term c and the coefficient term λ into the analysis. We make a reasonable assumption for the trading cost c, with R = 1.5% (representing the risk-free rate) and T = 1, resulting in c = 0.006%. The comparison between Figure 1 and Figure 2, displayed in the Appendix, vividly illustrates that the introduction of c and λ has smoothed the weights of the optimal portfolio, whether for the baseline or the 2-Step method, reducing spiky weight changes.

Furthermore, from Table 3, we can observe that the introduction of transaction costs not only smoothens optimal weights but also maintains favorable improvements of the 2-Step method. All of the 2-Step approaches enhance portfolio SR significantly, outperforming 'Equal_Vol' and 'Equal_Weight' portfolios, and effectively reduce portfolio volatility. While the smoothing of weights may lead to a slight reduction in SR and volatility compared to portfolios without cost penalties, it remains crucial for practical investment scenarios.

Table 3: Comparison of risk-return metrics between 2-Step and baseline methods with c = 0.006%.

	SR(2-Step)	SR	Std(2-Step)	Std
Cov	1.36	1.11	3.92%	4.18%
Elliptic_Envelope	1.41	1.34	3.89%	3.80%
Exp_Cov	1.33	1.15	3.96%	4.09%
LW_Const_Corr	1.51	1.48	3.56%	3.65%
LW_Regularized	1.41	1.14	3.77%	4.20%
LW_Single_Factor	1.50	1.47	3.58%	3.65%
Min_Cov_Det	1.40	1.34	3.88%	3.80%
OAS	1.45	1.14	3.90%	4.20%
Semi_Cov	1.50	1.47	3.62%	3.61%
Equal_Vol	NA	0.94	NA	5.09%
Equal_Weight	NA	0.78	NA	6.02%

4.1.3. Empirical Results of Max-Sharpe Optimizer

For simplicity, this section utilizes historical returns as expected returns within the Max-Sharpe objective function. Table 4 presents a comparative analysis of risk-return metrics between the 2-Step and baseline methods for an unconstrained Max-Sharpe problem with c = 0.006%. It is evident that the 2-Step method consistently demonstrates improvements relative to the baseline, much in the same vein as observed in previous Min-Var problem – characterized by an increased Sharpe ratio and a diminished portfolio volatility. Admittedly, using extrapolation of rolling historical returns as a proxy for expected returns is a straightforward approach, it may carry inherent risks, as the future always bring unforeseen events not reflected in historical data. Industry practices often involve more sophisticated modeling techniques for this purpose. However, delving into the intricacies of such modeling is beyond the scope of this paper.

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Table 4: Comparison of risk-return metrics between 2-Step and baseline methods with c = 0.006%.

	SR(2-Step)	SR	Std(2-Step)	Std
Cov	1.72	1.58	3.75%	3.74%
Elliptic_Envelope	1.66	1.42	3.77%	3.82%
Exp_Cov	1.72	1.65	3.75%	3.77%
LW_Const_Corr	1.72	1.58	3.70%	3.76%
LW_Regularized	1.58	1.55	3.69%	3.69%
LW_Single_Factor	1.67	1.57	3.75%	3.75%
Min_Cov_Det	1.64	1.46	3.80%	3.80%
OAS	1.59	1.56	3.67%	3.72%
Semi_Cov	1.64	1.66	3.73%	3.75%
Equal_Vol	NA	0.94	NA	5.09%
Equal_Weight	NA	0.78	NA	6.02%

4.1.4. Performance in Bearish Periods

In 2018 and the first half of 2022, the stock and commodity markets in China witnessed a bearish market condition, which typically pose challenges for portfolio investments, as strategies are likely to facing tail risks, and fund portfolios may encounter substantial drawdowns. Therefore, examining how the 2-Step method improves the risk measurement and the impact of introducing transaction costs on the cost-effectiveness of investment decisions during bear markets becomes crucial.

Table 5 presents the differences in various risk-return metrics between the 2-Step and baseline optimal portfolios when the cost is set at c = 0.006% in 2018. Evidently, even during bearish market conditions, the proposed approach continues to deliver significant improvements, enhancing portfolio returns while reducing portfolio volatility, downside volatility, and maximum drawdowns.

Table 5: Spread of risk-return metrics between 2-Step and baseline methods with the transaction cost c = 0.006%.

	Ret	Std	DStd	SR	Sortino	MDD
Cov	3.95%	-0.41%	-0.13%	1.02	1.48	-1.96%
Elliptic_Envelope	-0.31%	0.40%	0.86%	-0.17	-0.72	1.43%
Exp_Cov	3.44%	-0.66%	-0.75%	0.88	1.21	-2.47%
LW_Const_Corr	0.97%	-0.16%	0.29%	0.35	0.20	-0.05%
LW_Regularized	4.26%	-0.97%	-0.97%	1.25	2.12	-3.36%
LW_Single_Factor	1.20%	-0.19%	0.34%	0.44	0.28	-0.08%
Min_Cov_Det	0.00%	0.00%	0.10%	-0.00	-0.10	0.01%
OAS	3.96%	-1.01%	-0.80%	1.16	1.83	-3.27%
Semi_Cov	1.47%	-0.23%	0.28%	0.52	0.50	-0.38%
Average	2.10%	-0.36%	-0.09%	0.61	0.76	-1.12%

4.2. Limitations and Future Research Prospects

The research conducted in this article has its limitations, and there are several potential directions for future investigations. First, concerning the model inputs, this article uses hedge fund indices, which often comprise a multitude of constituent funds. Compared to actual portfolios, indices tend to have lower volatility, lower investment viability, and may contain missing data or outliers. Future research could consider using selected fund portfolios or incorporating investors' perspectives into model inputs. Second, there still exists several hyperparameters, such as α, λ, K, etc., which have not been thoroughly explored in this article. Investigating the sensitivity of the model to these hyperparameters is a potential direction for further research. Thirdly, this study employed a basic averaging method to calculate the short-term risk matrix. Future research may consider more sophisticated approaches, like Bayesian methods or machine learning, for their combination. Fourth, this study only applys two optimizers. In the academic literature, various other approaches have been proposed, including those by the references [4,6,7-8], or more complex objectives like Conic programming. Investigating the compatibility of the 2-Step method and the introduction of the cost term with them is worth exploring. Lastly, this study assumed a uniform transaction cost c for all assets. In practice, the trading frictions between assets can vary significantly (e.g., due to differences in liquidity or the use of over-the-counter derivatives). Therefore, personalizing c based on different asset characteristics and investors' perspectives is also an important direction for future research.

5. Conclusion

This article discusses the prevailing challenges in constructing hedge fund investment portfolios in the domestic market of China. Addressing these hurdles, this article employs the 2-Step Method within various covariance matrix estimation techniques proposed in academia and introduces a metric to assess the trading abrasion of hedge funds, innovatively incorporating both aspects into the fund portfolio optimization process, grounded on the Mean Variance Optimization framework. Subsequent empirical research is conducted utilizing six hedge fund indices sourced from the Go-Goal platform. This research, on one hand, clarifies that the 2-Step method effectively uses short-term risk estimations to construct various risk matrices and reduces the negative impacts of the non-normal return distribution on the accuracy of risk estimations. It also enhances the model's adaptability to short-term market regime-switching. As a result, the method substantially reduces portfolio volatility and improves the Sharpe ratio in comparison to baseline models. On the other hand, the introduction of transaction costs c and coefficient λ meticulously illustrates the funds' trading opportunity costs and significantly modulates the optimal portfolio's turnover rate, thereby refining the optimization process's alignment with real-world investment scenarios. Ultimately, by combining transaction cost control with the 2-Step method in both Min-Var and Max-Sharpe optimizers, there is a steady and long-term improvement in the portfolio's risk-return performance. This approach also shows significant resilience in multi-asset bearish market conditions, effectively reducing downside risks.

References

[1] Best, M.J. & Grauer, R.R. (1991). On the Sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results. The Review of Financial Studies, 4, 315-342.

[2] Chopra, V.K. & Ziemba, W.T. (1993). The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice. The Journal of Portfolio Management, 19(2), 6-11.

[3] DeMiguel, V., Garlappi, L., Nogales, F.J., & Uppal, R. (2009). A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms. Manag. Sci., 55, 798-812.

[4] Choueifaty, Y., & Coignard, Y. (2008). Toward Maximum Diversification. The Journal Of Portfolio Management, 35, 40-51.

[5] Qian, E.Y. (2011). Risk Parity and Diversification. The Journal of Investing, 20, 119-127.

[6] Rockafellar, R.T. & Uryasev, S. (2000). Optimization of Conditional Value-at-Risk. Journal of Risk, 2(3), 21-41.

[7] Estrada, J. (2007). Mean-Semivariance Optimization: A Heuristic Approach. Capital Markets: Asset Pricing & Valuation eJournal.

[8] Chekhlov, A., Uryasev, S., & Zabarankin, M. (2003). Drawdown Measure in Portfolio Optimization. Capital Markets: Asset Pricing & Valuation eJournal.

[9] Markowitz, H. (1952). Portfolio Selection. The Journal of Finance, 7(1), 77-91.

[10] Peter J. Rousseeuw (1984) Least Median of Squares Regression, Journal of the American Statistical Association, 79:388, 871-880

[11] Rousseeuw, P.J. & Van Driessen, K. (1999). A Fast Algorithm for the Minimum Covariance Determinant Estimator. Technometrics, 41(3), 212-223.

[12] Ledoit, O., & Wolf, M. (2003). Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. Journal of Empirical Finance, 10, 603-621.

[13] Ledoit, O. & Wolf, M. (2003). Honey, I Shrunk the Sample Covariance Matrix. The Journal of Portfolio Management, 30(4), 110-119.

[14] Ledoit, O., & Wolf, M. (2004). A well-conditioned estimator for large-dimensional covariance matrices. Journal of Multivariate Analysis, 88, 365-411.

[15] Estrada, J. (2006). Downside Risk in Practice. Microeconomic Theory eJournal.

[16] Arnott, R.D., Kalesnik, V., & Wu, L.J. (2023). Mitigating the Hidden Risks of Factor Investing. The Journal of Portfolio Management, Quantitative Special Issue 2023, 49(2), 111-124.

[17] Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 31(3), 307-327.

[18] Cornuéjols, G., Peña, J., & Tütüncü, R. (2006). Optimization Methods in Finance. Cambridge University Press.

Appendix

The portfolios optimal weights are displayed in this appendix. Figure 1 shows a comparison between optimal weights of the baseline and 2-Step method with fund trading costs (c=0.006%), while Figure 2 shows a comparison without fund trading costs.

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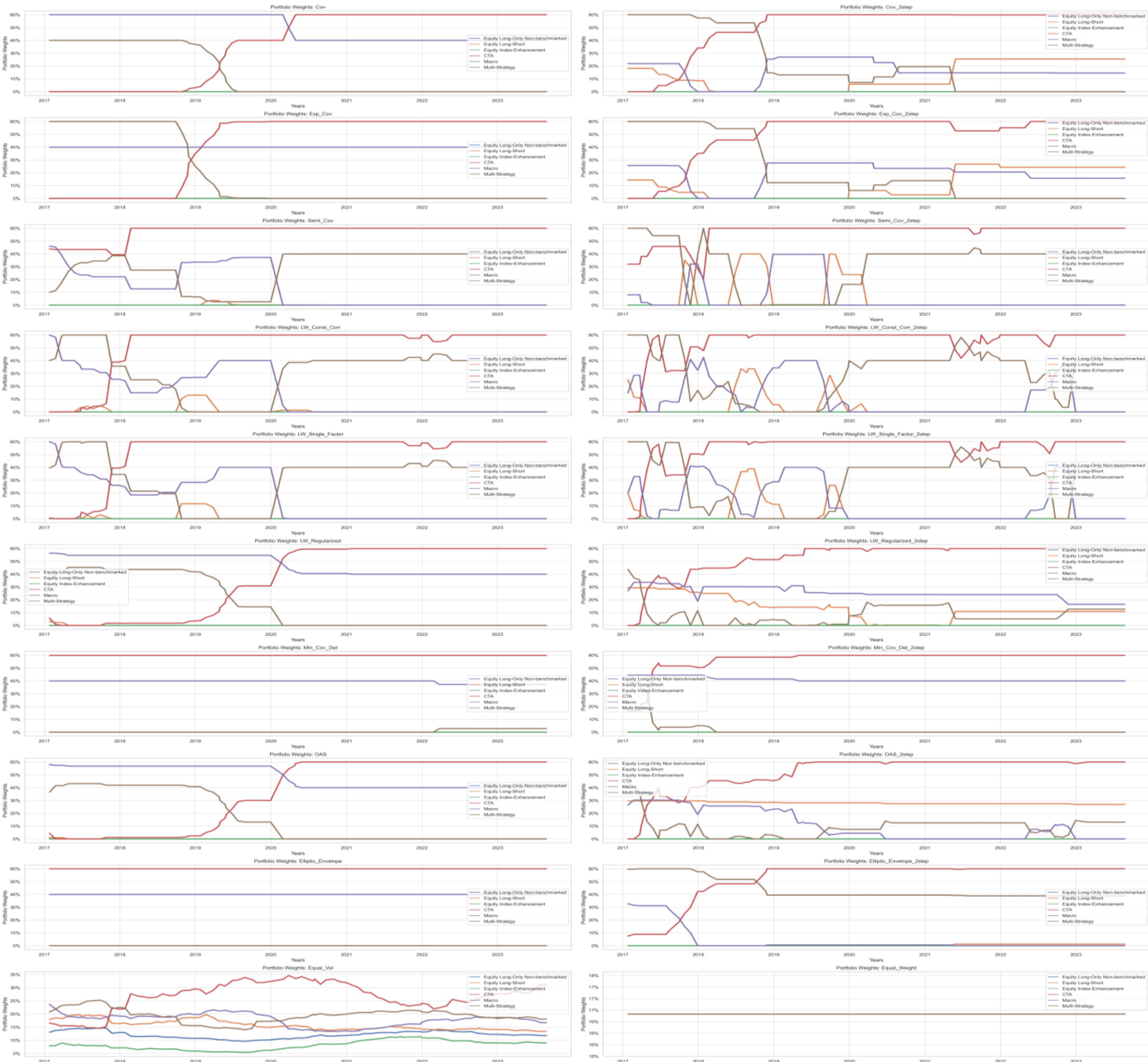

Figure 1: Comparison between optimal weights of the baseline and 2-Step method with the fund trading cost (c=0.006%)

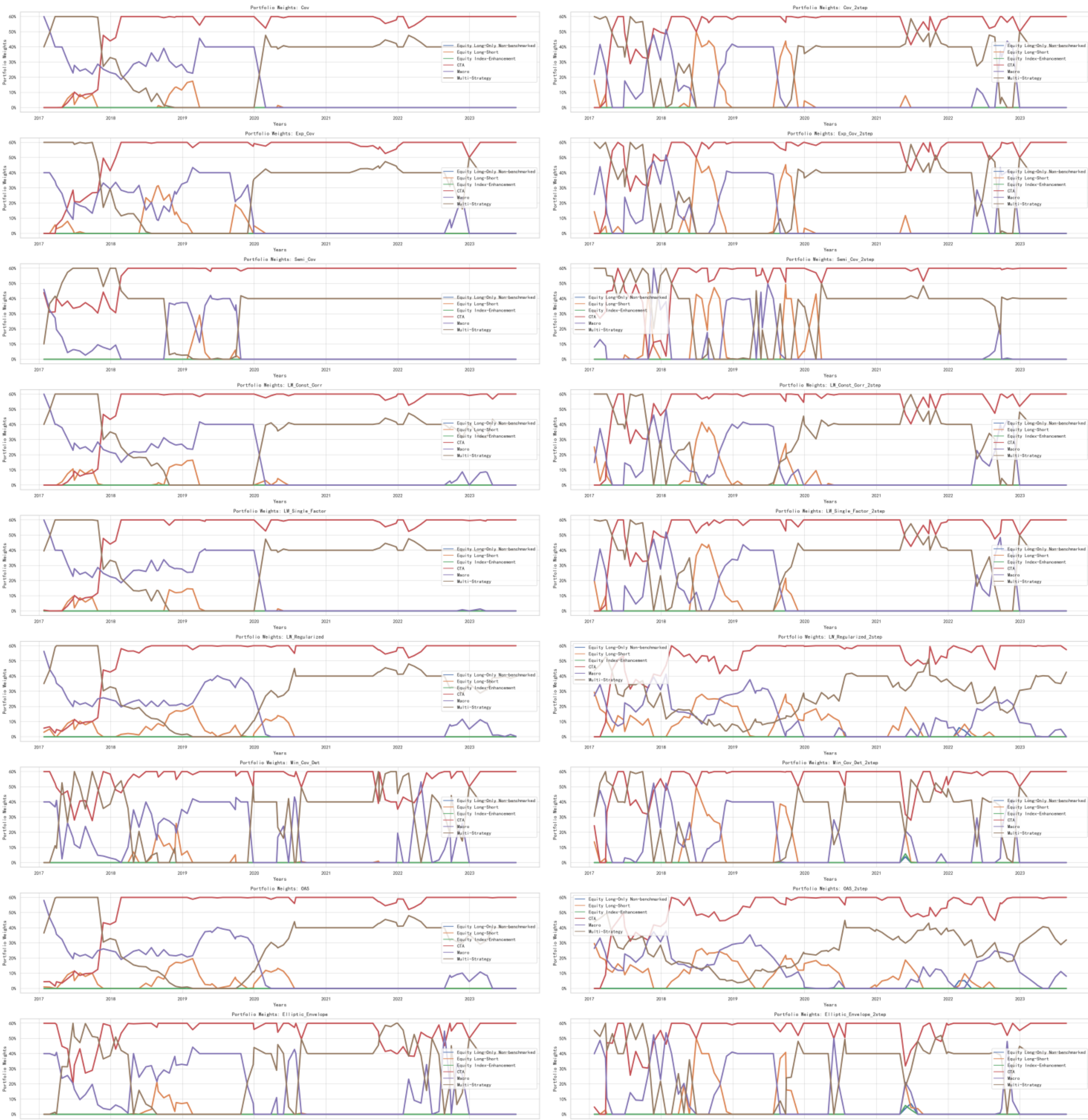

Figure 2: Comparison between optimal weights of the baseline and 2-Step method without the fund trading cost