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Mean-variance Portfolio Optimization by LSTM-based Predictions

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Abstract: With the progress of machine learning, its application fields are gradually increasing, especially in the field of quantitative finance, which is particularly outstanding, the Portfolio optimization combine with time series prediction and machine learning has brought great development prospects for investors. This study mainly employed the LSTM model and Mean-Variance Model to predict stock return and build an optimal combination respectively. This study selects relatively high weight stocks on the NASDAQ index, 'AAPL', 'AMZN',' ASML', 'AVGO', 'GOOG', from December 31, 2019, to July 1, 2023. First, the study obtained the predicted stock prices of five stocks through LSTM Model and based calculated the predicted returns on a rolling basis. Second, based on the modern investment theory, this study uses the predicted rate of return to construct the optimal daily investment ratio through Mean-Variance Model. Finally, this study compared cumulative return of optimal portfolios with the NASDAQ within the same time frame. This study draws a conclusion that hybrid model which combine the stock price forecasting with asset allocation can indeed bring excess returns.

Keywords: Mean-variance model, portfolio management, LSTM model

1. Introduction

The Markowitz mean variance (MV) model, developed in 1952, is the foundation of portfolio theory. The MV model discussed problems mainly focus on the mean return of portfolio, and the variance or standard deviation problem of the portfolio return [1]. The portfolio generated by this model can simultaneously considered to achieve the expected return and reduce the risk of the investment [2]. For a given expected return, the effective frontier formed by MV model can provide the optimal investment strategy [3].

A deficiency of traditional MV models is that this model input historical data and use it to study the optimal asset allocation in the future, it will cause the MV model to receive incorrect input values, resulting in the extreme positive and negative weights [4]. Reasonable and accurate predictions can effectively help investors achieve higher investment returns and achieve reasonable risk hedging [5]. Therefore, it will be beneficial for investors to incorporate stock price prediction models in the process of portfolio optimization [2].

Since all public information is reflected in the stock price, the random new information will lead to the price trend exhibits randomness, therefore, the stock price behaves as a "random walk", making it difficult for investors to defeat the market [6]. Research has found that the return of individual

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stocks and investment portfolios using simple neural networks are higher than those established using traditional statistical methods, it is mainly because the neural networks can flexibly capture dynamic and interacting variables [7]. Long-short-term memory network (LSTM) as one of excellent RNN model because concept of 'gates' was introduced, it can learn memory of short-term and support memory of long-term [8]. This study proposes a hybrid model that combines the LSTM with the MV model, and the results show that this model can bring excess returns.

This paper is structured as follows: the next section is five stocks selected for this study and conducted descriptive statistics, section 3 introduces the methodology which including Long-short-term memory network (LSTM) and MV Model. After that, in section 4, this study scrutinizes the efficacy of the proposed approach and compare with the benchmark Index. The study result will help the investor and portfolio manager to realize the outstanding performance of combining machine learning and portfolio management.

2. Data and Methods

2.1. Data

This study employee five stocks with relatively high proportion in the NASDAQ index to construct an investment portfolio and obtain excess returns compared to the NASDAQ index. The daily stock data used in this study are collected from Yahoo finance (https://finance.yahoo.com/). The ticker of the 5 stocks is AAPL, AMZN, ASML, AVGO, GOOG. The data range is from December 31, 2019 to July 1, 2023. The basic variables involved are date, High price, Adj Close price, Low price, open price, Volume as well as the processing variable is daily return.

This study uses the adj_close prices to calculate the daily log return and obtained the distribution histogram of daily returns in Figure 1.

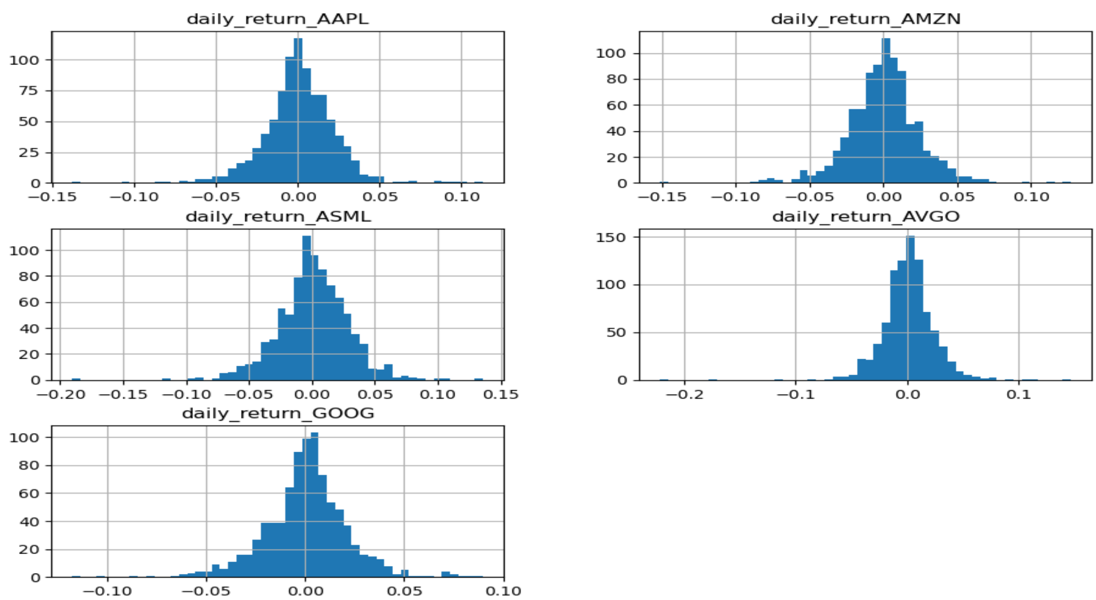

Figure 1: Daily returns distribution

Based on the daily return, the following basic informations were calculated and showed in the following Table 1.

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Table 1: Descriptive statistics of five stocks daily_return

	AAPL	AMZN	ASML	AVGO	GOOG.
Mean	0.0011	0.0004	0.0011	0.0013	0.0007
Std	0.0221	0.0243	0.0282	0.0248	0.0216
Skew	-0.1385	-0.1354	-0.2904	-0.8637	-0.1868
Kurt	4.5211	3.7640	3.5199	12.5544	3.0119

2.2. Method

Time series data analysis plays a crucial role in financial learning and prediction. Stock investment with high volatility is usually considered as high-risk product. If prices of stock or the return of stock can be effectively predicted during the investment process and combined with reasonable asset allocation strategies, investors will obtain higher returns. This study attempts to combine machine learning with portfolio optimal asset allocation. which mainly consists of the following four steps (See Figure 2).

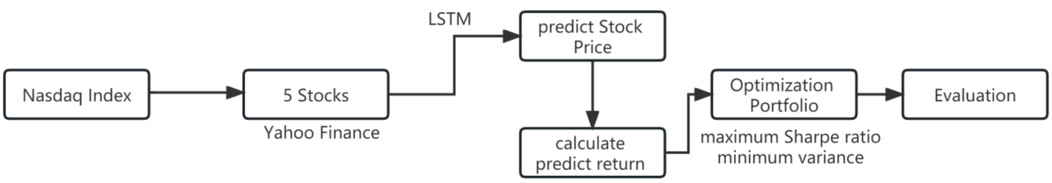

Figure 2: Study process

Firstly, this study selected five stocks of the Nasdaq Index and Download basic data from Yahoo finance then processed to obtain daily log return data. Secondly, this study's data spans the period from December 31, 2019, to July 1, 2023. In order to forecast the adj_close stock price on a rolling basis, the LSTM neural network uses the first 80% of the stock data as the train set, the last 20% of the stock data as the test set to forecast the adj_close stock price on a rolling basis. After that daily anticipated returns were computed based on forecasting adj_close stock prices. Thirdly, using the mean-variance optimization method and the daily return predicted in part 2 as a starting point, this study determines the ideal portfolio weight for each day. and build two models, the highest Sharpe ratio and the lowest variance, in order to determine the optimal portfolio weights for each day. then combine the daily actual asset return rate with this information to calculate the portfolio return rate. Finally, compute the cumulative returns of the two models over the testing period and compare them to the Nasdaq Index's cumulative returns.

2.2.1. Long Short-Term Memory

With the boom in artificial intelligence technologies, research into the machine learning which used in the quantitative investments is increasing, especially using machine learning to predict the price or return trend of individual assets.

Considering the non-linear relationship in stock prices, neural networks are a suitable model to predict the price of stock [9]. Therefore, neural networks are more suitable for handling nonlinear relationships in stock prices. LSTM is one of the most successful RNNs. RNN networks are only suitable for learning short-term memory, and this model does not support learning for long sequence memory [10]. LSTM is particularly suitable for time series prediction. In terms of long-term memory, it can effectively "learn" and "remember" market mechanisms and other things (See Figure 3). In terms of memory of short-term and review window, interaction ensures stable performance in prediction of short-term [8].

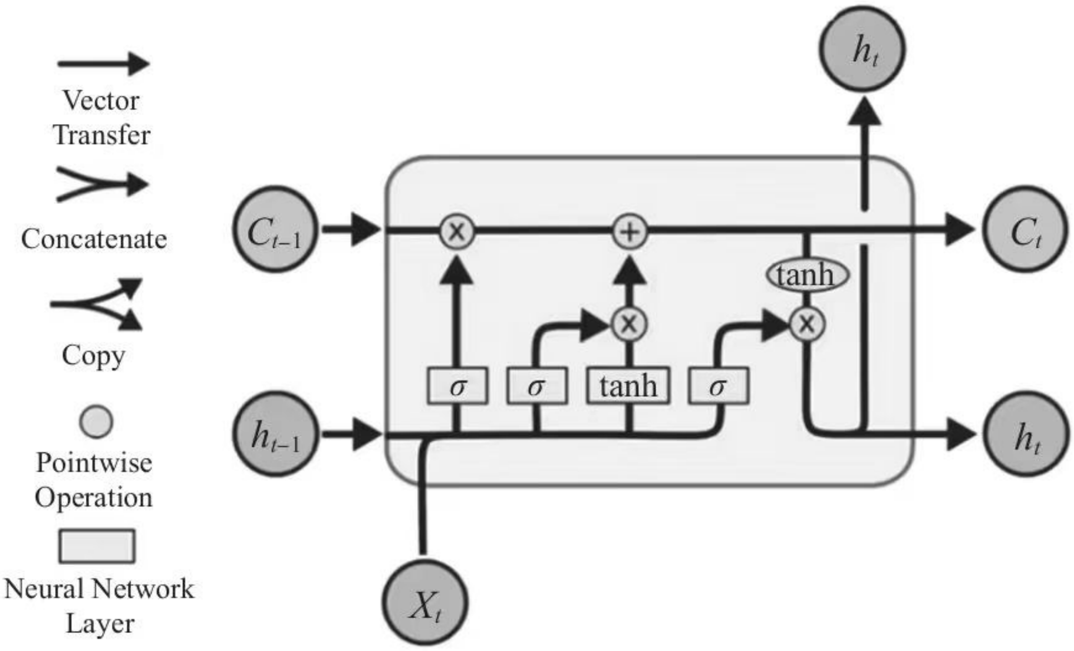

Figure 3: LSTM Model

Three types of gates are used in an LSTM network. The first one is the forget gates, determining which interference information is discarded in the cell state. which achieved by reading ht-1, xt, and giving different weights between [0,1], where 1 represents complete acceptance and 0 represents complete discarded [10]. The second one is input gates, determining the degree of information update and determining the amount of updated content based on the sigmoid functions [10]. Discard abandoned information in the forget gates and add updated information, storing it in a long-term state. The third one is output gates, determining the output level of sequence information. The output part of the cell state is determined by the sigmoid layer, and the long-term state of the cell is done using the tanh function.[10].

In order to measure error or the difference between the actual adj_close price and the predict adj_close price, this study employee the MSE which is the mean/average of the square of all of the error, The formula is as follows:

$$\mathrm{MSE} = \frac{1}{\mathrm{n}} \sum_{\mathrm{i}}^{\mathrm{n}} w_i (\mathrm{Y}_i - \hat{\mathrm{Y}}_i)^2 \tag{1}$$

Where n is the number of assets, Yi is the actual adj_close price of each asset, $\hat{\mathrm{Y}}_i$ is adj_close price of each asset.

2.2.2. Mean-Variance Model

Modern Portfolio Theory (MPT) is the portfolio optimization framework developed by Harry Markowitz in 1952.The optimization process used in MPT is often referred as Mean-Variance

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Optimization (MVO), which aims at helping investors optimize their portfolio by balancing risk and return [11].

In Harry Markowitz's investment theory, portfolio variance and Weighted average return are used to define portfolio risk and portfolio return. The formula is as follows:

$$\mathrm{E}(R_P) = W^T R = \sum_i w_i r_i \tag{2}$$

Where $\mathrm{R_P}$ is the Portfolio return, $\mathrm{W_i}$ is the proportion of $\mathrm{Stock_i}$ in the portfolio group, $\mathrm{r_i}$ is the return of $\mathrm{Stock_i}$. In this study, this study uses the i-day real return of each asset and the optimal asset proportion which calculated based on the i-day predicted return rate to calculate the portfolio return.

$$\sigma_P^2 = \mathrm{var}(\sum_i w_i r_i\ ) = \sum_{ij} w_i\, w_j \mathrm{cov}(r_i r_j) \tag{3}$$

Where σ_P^2 is the variance of Portfolio, $\mathrm{cov}(\mathrm{r_i r_j})$ is the covariance of the returns of Stock_i and Stock_j. In this study, in order to ensure maximum utilization of funds and not consider short selling stocks, we added two constraint conditions, $\sum_\mathrm{i} \mathrm{w_i} = 1$ and $\mathrm{w_i} \geq\ 0$. This study uses the real daily returns of each asset from day 1 to day i-1 to construct covariance matrix on a roiling basis.

In addition, the Sharpe ratio is a popular indicator for evaluating risk adjusted returns, The formula is as follows:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p} \tag{4}$$

where$\mathrm{R_P}$ is the Portfolio return, R_f is the current risk-free rate of the market. σ_p is the standard deviation of the portfolio's return. In this study, assume R_f is 4.2%, which is the yield of ten-year treasury bond.

This study considers that investors have varying levels of risk tolerance and then selects the maximum Sharpe ratio and minimum variance as two indicators to construct two models to confirm the optimal daily investment ratio.

3. Results

This study uses the LSTM model to predict adj_close prices on roiling basis. After debugging the model parameters, the final LSTM model consists of two input layers, each with 70 units, dropout rates are 0.2. and the model trained for 50 epochs. The evaluation indicators R^2 of the mode are 0.9371, 0.9508, 0.8848, 0.9253 and 0.9538 for AAPL, AMZN, ASML, AVGO, GOOG respectively. And the following Figure 4 shows the prediction results of the LSTM model.

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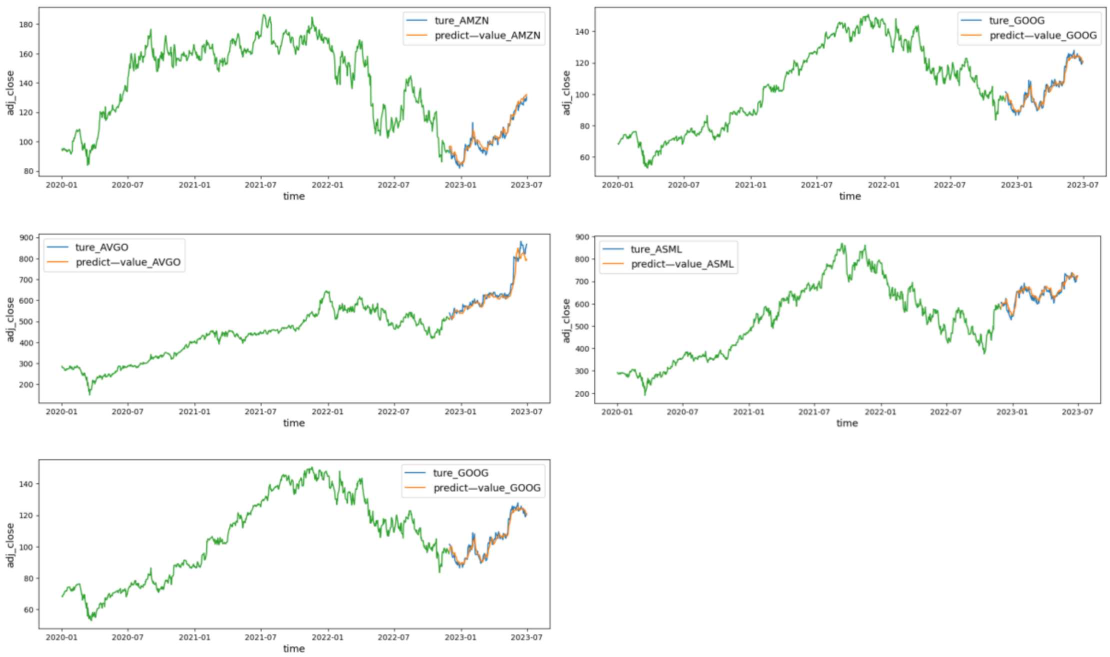

Figure 4: LSTM performance of adj_close price priediction

Overall, the model predicts the future value of asset prices to a certain extent, indicating a close consistency between the predicted values and the test set (See Table 2).

Table2: Evaluation indicators for prediction results

	APPL	AMZN	ASML	AVGO	GOOG
R2	0.9371	0.9508	0.8848	0.9253	0.9538
MAE	3.7848	2.2773	13.1465	17.8043	2.0318
MSE	18.6975	7.9101	262.5134	665.3175	6.7330
RMSE	4.3241	2.8125	16.2023	25.7937	2.5948

This study proposes trade on a daily basis. During testing period, the daily asset weight is determined by the daily stock price predicted by the LSTM model. According to the Portfolio Theory, this study has chosen two allocation methods. The first is the maximum Sharpe ratio model, based on the maximum Sharpe ratio of the daily portfolios to determine the proportion of assets. and the second is the minimum variance model which based on based on the minimum variance of the daily portfolios.

This study compared the cumulative returns of the two models and the Nasdaq Index over the test interval. In the maximum Sharpe ratio model, the cumulative return during the period is 37.0460%, and the volatility is 0.146190. In the minimum variance model, the cumulative return during the period is 29.0335%, and the volatility is 0.118845. The cumulative return of the Nasdaq Index during the same period is 18.8957%, and the volatility is 0.073122 (See Figure 5 and Table 3).

Table 3: Cumulative yields and volatility

Portfolio	cumulative return	volatility
maximum Sharpe ratio model	37.05%	0.14619
minimum variance model	29.03%	0.118845
Nasdaq Index	18.90%	0.073122

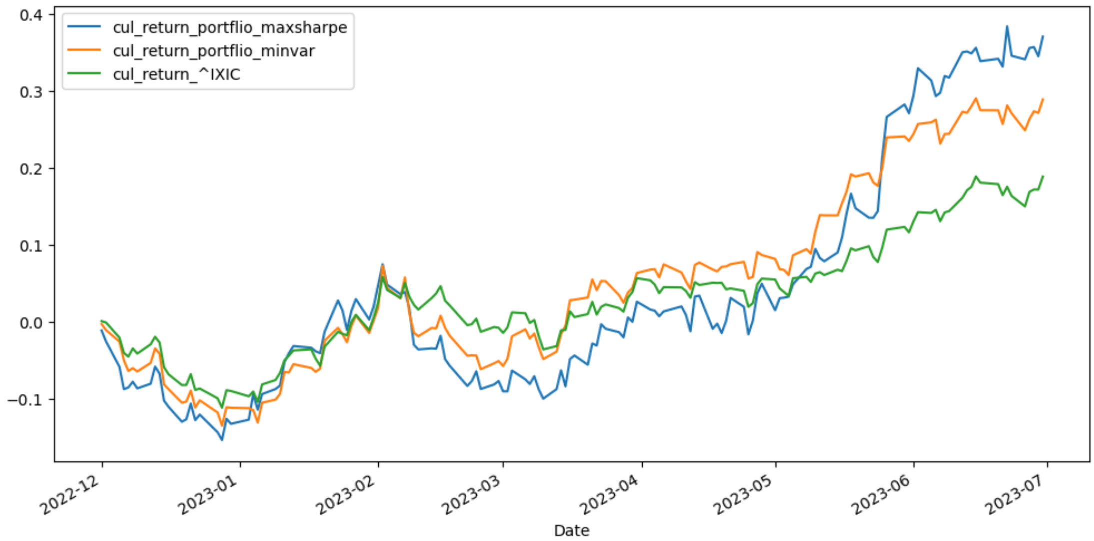

Figure 5: Cumulative Returns of two models and IXIC index

The green line represents the cumulative return of the Nasdaq Index. Obviously, both models outperformed the index, and in terms of cumulative returns, the maximum Sharpe ratio model showed the best results.

4. Conclusion

This study selects five stocks from the NASDAQ index: 'AAPL', 'AMZN', 'ASML', 'AVGO', 'GOOG', from December 31, 2019, to July 1, 2023. A new approach to structure portfolio is based on the LSTM model for prediction of stock price and the mean deviation (MV) model for portfolio selection. First, this study uses LSTM model to predict the daily price and calculate the predict daily return. Second using MV model to construct two model: Maximum Sharpe and Minimum Variance, based on the daily predict return. Finally, compare the cumulative return of two model with the Nasdaq Index's cumulative returns. The results indicate that combining the LSTM stock price prediction model can provide a more effective portfolio asset allocation strategy. However, this study did not fully consider transaction costs and the introduction of leverage. Therefore, in future real stock allocation process, the model proposed by this research institute can be further optimized and adjusted to obtain better results.

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