Research on the Comparison of Sampling with and Without Replacement in Quality Control Test

Yilin Yang[1,a,*]

[1] *Chinese University of Hong Kong, Shatin NT, Hong Kong SAR, China*
a. elynyang@link.cuhk.edu.hk
**corresponding author*

Abstract: The choice of a reasonable and scientific sampling method and the implementation of efficient quality control tests can help companies achieve economic and social benefits. In this paper, probability theory and statistics are applied to provide a basis for enterprises to consider the choice of sampling with and without replacement through case studies and modeling. These two sampling methods lead to binomial and hypergeometric distributions of probabilities, respectively. The differences between them are found to be negligible in the case of large population size by calculations at $N \to \infty$. Based on comparison of sampling methods, this paper further divides the manufacturing industry into the chemical industry, which is sensitive to the reliability of quality control test, and other manufacturing industries, which are more sensitive to cost while controlling quality. In the end, suggestions are proposed for improving quality control tests for the category industries respectively.

Keywords: binominal distribution, hypergeometric distribution, sampling method, quality control test

1. Introduction

Sampling and testing of products for quality control is an important step in the process from production to market. Whether quality control test is implemented effectively can directly affect the economic and social benefits of an enterprise. In quality control test, a reasonable and scientific sampling test can help companies save costs while gathering information on product quality. Therefore, how to choose an appropriate sampling scheme and how to improve quality control test is an essential issue that companies must address.

A large number of studies have been conducted using probability theory and statistical knowledge to provide suggestions for enterprises to improve their sampling and testing programs. In the pursuit of profit maximization, scholars are constantly optimizing the sampling scheme with the goal of reducing the required sample size. Studies have ranged from using Bayes' principle to determine the maximum entropy prior distribution and thus reduce the minimum sample size for sampling and inspection schemes [1];to using conjugate prior distributions to further reduce the sample size requirement under maximum risk taking [2]; in the case of known defect rates, it has been demonstrated that the calculation of inventory sampling quantities can be simplified by replacing the hypergeometric distribution with a binomial distribution, thereby reducing inspection costs [3]. More theoretical studies have also obtained equiprobability formulas for hypergeometric distributions by suitable sub-sampling designs and applied [4]; a stochastic mathematical model is used to

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demonstrate that an economically designed sampling plan can result in 20% cost savings [5]. There are also studies focused on a specific industry, such as the use of secondary sampling in the textile industry to ensure the interests of both manufacturers and merchants [6], the use of the "σ" method to design a more efficient sampling scheme in the machinery industry, where the number of products to be inspected is large and the accuracy of inspection is high [7], and so on.

The research is not limited to sampling schemes, but also focuses on the entire quality control testing process, expecting to improve the efficiency of quality control test by standardizing and monitoring the operation of all aspects of the testing process [8]. It has also been pointed out that when evaluating the results of sampling tests, the evaluation system needs to be targeted according to the characteristics of the sample [9]. For chemical industry, in addition to focusing on the operational process of sampling and testing and the professionalism of staff [10], attention needs to be paid to the maintenance of testing instruments and the safety of the entire testing process [11].

In this paper, the binomial distribution and hypergeometric distribution models are established for sampling with and without replacement in product quality control test, respectively. Case study is used to compare and study the applicability of the two sampling schemes; then the connection between the probability distributions of the two different models in the case of large population size is further discussed; finally, suggestions for optimizing quality control test are given according to the characteristics of different industries. This paper focuses on the model comparison and linkage analysis of different sampling schemes, and further discusses the improvement of whole process of quality control test, with the expectation to provide ideas for enterprises to optimize quality control test.

The main contribution of this paper is to combine the relevant content of probability theory and statistics with different sampling methods to provide a basis for the choice of sampling methods when companies conduct quality control test. Also, according to characteristics of different industries, recommendations are given to improve the quality control test. These recommendations provide ideas for enterprises to improve the efficiency of quality control test and reduce costs.

2. Analysis of Product Quality Control Testing Based on Sampling with and Without Replacement

There are two methods of quality control tests to be compared. The difference is sampling with or without replacement, which leads to different distribution functions of probability. To compare these two sampling methods in quality control test, a case of Company A is given as below.

According to National Bureau of Statistics of China, 2021 China manufacturing product qualification rate is 93.86%. Considering that the product qualification rate is usually slightly higher than national average, it is assumed that the production qualification rate in Company A is 94%. Accordingly, the probability of producing defectives (p) in Company A is 6%.

Then use RANDBETWEEN in Excel to produce population size (N) and sample size (n). To get the population size, type in RANDBETWEEN (500,1000) and 650 is given; to get the sample size, type in RANDBETWEEN (30,90) and 65 is given.

So here is a case which assumes Company A has a probability of producing defectives p=6%, a population size of N=650, and a sample size for quality control test of n=65.

2.1. Description of Sampling Without Replacement

The first sampling method for quality control test is to take a sample of 65 one by one from the total population, which is a sampling without replacement method with probabilities that follow a hypergeometric distribution of the discrete distribution. In hypergeometric distribution, M samples

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out of *N* has a certain attribute, which is being defectives in this case. *M=p(N)=6%(650)=39*. To conclude, *X* follows a distribution of *X~H (65,39,650)*.

The probability of finding k defectives is:

$$P(X=k)=\frac{C_M^k C_{N-M}^{n-k}}{C_N^n}=\frac{C_{39}^k C_{26}^{65-k}}{C_{650}^{65}}, k=0,1,2,\dots,39 \quad (1)$$

The probability of finding defectives is given by:

$$P\sum_{k=1}^{39} P\ (X=k)\ =\frac{\sum_{k=1}^{39} C_{39}^k C_{26}^{65-k}}{C_{650}^{65}}=0.9856 \quad (2)$$

Therefore, in this method of sampling, there is a probability of 0.9856 to find defectives.

2.2. Description of Sampling with Replacement

The other sampling method of quality control test is to take one sample out of the population and do the quality test, then return this sample and take another. Repeat above procedure for 65 times. For every test, there are only two possible results, which are "qualified" and "not qualified". This creates a Bernoulli experiment. Since the result of every test is independent, repeating such procedure is doing independent repeat experiment. As a result, the probability of this sampling with replacement method follows a binominal distribution.

In the same Company A case, with the sample size of n=65, probability of producing defectives p=6%, X~B (65,6%) is given.

The probability of finding *k* defectives is:

$$P(X=k)=\left(C_{65}^k\right)6\%^k(1-6\%)^{65-k}, k=0,1,2,\dots,39 \quad (3)$$

The probability of finding defectives is given by:

$$\sum_{k=1}^{39} P\ (X=k)\ =\sum_{k=1}^{39}\left(C_{65}^k\right)6\%^k(1-6\%)^{65-k}=0.9821 \quad (4)$$

When sampling with replacement for 65 times, there is a probability of 0.9821 to find defectives.

2.3. Comparison

2.3.1. Comparison of Calculation Results

In Company A case, there are two methods of sampling for quality control test mentioned above. One is taking a sample of 65 one by one from the total population, which leads to a hypergeometric distribution of probability; the other is to repeat the procedure of "take out- return" for 65 times and results in a binominal distribution of probability. After calculating these two distribution models, probabilities of 0.9856 and 0.9821 are given respectively. And 0.9856 is higher than 0.9821, which means sampling without replacement creates a higher probability of finding defectives.

When a company is doing quality control test, it is usually expected that the probability of detecting defectives should be as high as possible. Only in this way can a company achieve its goal of quality control with efficiently. Taking the aim of quality control test into consideration, the first method of sampling without replacement is of higher efficiency.

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2.3.2. Comparison of Real-world Applications

Accuracy of quality control test

Apart from exact probabilities of two different sampling methods, the stability of probability of every draw is worth considering as well. Obviously, when sampling without replacement, the population size decrease with every draw. And this is the cause of increasing probabilities in finding defectives of every draw. In other words, although the probability of sampling without replacement is higher, it changes with every draw and results in an unstable probability.

In contrast, sampling with replacement guarantees a stable probability. Because it returns every sample back to sample lot after testing, sample size remains the same in each draw, and thus ensuring a stable probability.

In quality control tests, the stability of every draw is necessary, which is expected to remain the same to ensure the accuracy of every test. From the perspective of this point, sampling with replacement can better ensure the accuracy of test.

Cost Control

In the case of Company A, there is limited information of tested products. However, when applying it into real life, features of products tested also matter. For different industries, quality control tests may bring damage to products on different level. Take vehicle industry as example. A crash test is a destructive but necessary test in quality control, which ensures safe design of vehicles. It is also costly since it causes irreversible damage to vehicle tested.

To maximin profit, it is usually unwise for a company to replace the damaged sample with a new one. Apparently, if quality control test brings irreversible damage to product, only sampling without replacement can be applied.

3. Further Discussion

In Company A case, the probability of detecting defectives in two methods are 0.9856 and 0.9821 respectively. Given that difference between them is slight, this paper further discusses the connection between hypergeometric distribution and binominal distribution.

With population size N becoming large enough, defective probability $p = \lim_{N\to\infty} \frac{M}{N}$,

$$\frac{C_M^k C_{N-M}^{n-k}}{C_N^n} = \frac{M!}{k!(M-k)!}\left\{\frac{(N-M)!}{(N-k)!(N-M-n+k)}\right\}\left\{\frac{n!(N-n)!}{N!}\right\} =$$

$$\frac{n!}{k!(n-k)!}\left\{\frac{M(M-1)\dots(M-k+1)}{N^k}\right\}\left\{\frac{N^n}{N(N-1)\dots(N-n+1)}\right\}\left\{\frac{(N-M)(N-M+1)\dots[N-M-(n-k)+1]}{N^{(n-k)}}\right\} \quad (5)$$

With certain *n* and *k*,

$$\lim_{N\to\infty} \frac{M(M-1)\dots(M-k+1)}{N^k} = p^k \quad (6)$$

$$\lim_{N\to\infty} \frac{N^n}{N(N-1)\dots(N-n+1)} = 1 \quad (7)$$

$$\lim_{N\to\infty} \frac{(N-M)(N-M+1)\dots[N-M-(n-k)+1]}{N^{(n-k)}} = (1-p)^{n-k} \quad (8)$$

Thus,

$$\lim_{N\to\infty} \frac{C_M^k C_{N-M}^{n-k}}{C_N^n} = C_n^k p^k (1-p)^{n-k} \quad (9)$$

The above procedure demonstrates that the hypergeometric and binomial distributions are the same when the population size becomes infinite ($N \to \infty$). It explains the slight difference in probabilities between two different sampling methods in Company A case. In this case, it is reasonable to predict that if population size becomes much larger than 650, difference between two sampling methods will eventually decrease to 0.

This conclusion can be applied to real-world cases when population size of every lot is large enough. In this situation, samples are taken from the population for quality control test, and whether they are returned or not will have no effect on testing results. This is because the few samples taken are negligible for the large population.

Therefore, in the real production process, if population size is large enough, difference between the sampling with and without replacement can be ignored, and the choice of sampling method will have no effect on the quality control results.

4. Suggestions on Improving Quality Control Test

As it has already been discussed, product quality control test in real life needs to take into account the unique characteristics of products in different industries. There are different issues need to be noted for products in different industries.

The following classifies industries into two categories based on their product characteristics. Among these product characteristics, quality control reliability and quality control cost are two major factors to classify industries. Finally, two categories of industries are obtained. One category is chemical industry which requires high product quality control reliability and is not as sensitive to cost as other manufacturing industry. The other category is general manufacturing industry which is more sensitive to quality control cost. Suggestions will be given separately to improve quality control test.

4.1. Chemical Industry

In chemical industry, the reliability of quality control test means both accuracy and safety. Chemical products are significantly different in characteristics from other general manufacturing products, and the accuracy of quality testing has a strong impact on the safety of chemical products. For example, the content of chemical components and the concentration of acids and bases in chemical products can affect safety performance of products. Therefore, chemical products need to undergo strict quality testing before entering the market. On the other hand, due to the hazardous nature of chemical products, safety is also an issue that needs to be emphasized in quality control test.

4.1.1. Develop a Scientific Sampling Program

Chemical products in the quality testing sampling can not simply consider sampling with or without replacement issue, but to develop a scientific sampling program for certain product characteristics. Some of the chemical products may be unstable, corrosive, flammable or explosive, so it is important to ensure the safety and accuracy of sampling program. In addition to the sampling method, the program should also include safety measures, sampling tools, sample transportation and preservation measures. Among them, safety measures guarantee the safety of sampling personnel when facing hazardous chemical products or emergency; others is to assure that the chemical substances do not deteriorate, thus ensuring the reliability of the quality test results.

4.1.2. Improve the Management of Sampling Personnel

Due to the hazardous nature of chemical products, sampling personnel need to make sure that they have read the sampling program in full and thoroughly understand the characteristics of samples in

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advance and take protective measures according to the product characteristics. Implementing sampling and testing in strict accordance with the sampling program during the implementation process is also important.

After the completion of sampling, sampling personnel need to do a standard sampling record. For chemical products, sample name, sampling date, sample properties, etc. are critical factors that will affect the quality test results, which need to be recorded completely without errors by the sampling personnel.

4.1.3. Maintenance of Quality Testing Devices

In addition to scientific testing programs and professional testing personnel, quality testing of products in the chemical industry also has requirements for testing devices. The accuracy of the testing devices directly affects the outcome of tests. Hence, testing devices need to be regularly checked and maintained so that they always perform in accordance with the testing standards. This will assure the accuracy of the quality test results.

4.2. General Manufacturing Industry

Compared to chemical industry, general manufacturing industry do not require extremely high accuracy of quality inspection. In contrast, traditional manufacturing industries are more concerned with cost control, and their goal is to accomplish quality control test while spending relatively little money.

4.2.1. Control of Sample Size

Control of sample size is significant to quality control test in general manufacturing industry. When the sample size is too large, quality testing will waste too much cost; when the sample size is too small, quality testing can not ensure accuracy. It will cause sampling errors so that the sample does not represent overall level of product quality. For this reason, regardless of whether to choose sampling with or without replacement, it is necessary to establish a reasonable and scientific sample size based on the reliability requirements specified by the state, achieving aims of both cost control and sampling accuracy.

4.2.2. Reasonable Definition of Testing Standards

Unlike products in the chemical industry, many products in the general manufacturing industry are difficult determine whether they are qualified through a specific test value. On the one hand, it is difficult to quantify product quality testing results. For example, in quality inspection of garments, it is difficult to quantitatively assess whether the alignment of a garment is qualified with a number; on the other hand, general manufacturing products often have multiple indicators to measure product quality. For example, in quality inspection of food, the taste, appearance, and nutritional value of food are all factors to consider. Imperfect testing standards in this case can cause a lot of wasted costs.

As a result, the criteria for product qualification need to be scientifically defined. When quality control test results are difficult to quantify, they can be finely classified, such as superior products, inferior products, unqualified products, etc.; when quality control test results need to consider several factors, it is necessary to develop reasonable quality inspection standards according to the main performance of the product.

5. Conclusion

In this paper, the difference between sampling with and without replacement in product quality control test is demonstrated in a comparative case study. It is found that in the case of a small population size, sampling with replacement produces a binomial distribution and sampling without replacement produces a hypergeometric distribution. In the case of a given population size, sample size and defective probability, sampling without replacement has a higher probability of finding defectives. Therefore, sampling without replacement satisfies the original purpose of a company's product quality inspection, that is, the higher the probability of finding defective products, the better. In practice, the choice of sampling scheme also requires consideration to ensure the accuracy of quality control test and cost control. Sampling with replacement ensures the stability of every draw, which makes quality control test more accurate; sampling without replacement is a more cost-effective way of sampling when product is irreversible damaged by quality test.

However, in the case of large population size, hypergeometric distribution is equal to binomial distribution, and the difference between putting back the sample or not can be ignored with respect to the total. Therefore, the choice of sampling method will have no effect on the quality control test results.

Based on the comparison of sampling methods, this paper divides the industry into chemical industry and general manufacturing industry on the basis of their sensitivity to two factors: quality control reliability and quality control cost. For the chemical industry, developing a scientific sampling program, improving the management of sampling personnel and maintenance of quality testing devices are used to ensure the accuracy and safety of quality testing; with the goal being to accomplish quality control test while spending relatively little money, general manufacturing industries can use methods of controlling sample size and reasonable definition of testing standards to improve quality control test.

However, there are influential factors that are not included when the study is conducted by classified industry. Future research can further do a good job of industry segmentation and propose optimized solutions for quality control test in a more targeted manner.

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