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Option Pricing and Risk Hedging in the Current Financial Market: A Case for Google

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Abstract: In the past few decades, financial derivative securities have been developing rapidly around the world, and the issue of options and investment consumption has attracted more and more attention from mathematicians and financiers at home and abroad. In this paper, option pricing models are constructed and calibrated based on the Black Scholes Merton model, binomial tree model, historical data model and Monte Carlo diffusion model. The differences between different option pricing models for options and stock hedging of the same company in a short period of time are discussed and analyzed. In this article, the Monte Carlo model outperforms the traditional Black Scholes Merton model, while the binomial tree model and the historical data model do not perform well. The results of this paper are beneficial for investors to use the optimal model to predict option prices, weaken the aggregate risk, and improve the aggregate return level.

Keywords: Google, BS, option pricing

1. Introduction

Consistent with historical trends, September 2022 is shaping up to be a tough month for U.S. equities. The major U.S. benchmark stock indexes have fallen more than 2% cumulatively so far this month, and the Federal Reserve is expected to raise interest rates sharply again later this week. Even strong technology giants such as Google, the company's stock price is also down in this shock streak. Therefore, it is important to choose the right option pricing model and a reasonable hedging strategy to help reduce risk and increase returns.

The theory of option pricing was first proposed by the French economist Bachelier, who first introduced the problem of option pricing in an article in 1900 and was later supplemented by Boness [1]. The Black-Scholes model is a more ideal European option pricing model, and the model laid the foundation for the development of options, which is of great significance in theory and practice [2].

In the field of quantitative finance, a Monte Carlo option model employs Monte Carlo techniques to determine the value of an option with numerous sources of uncertainty or complex features [3]. In this article, the Monte Carlo model, Black Scholes Merton model, binomial tree model, the historical data model, delta hedging, and calibration are the main components. With the data collected from Alphabet Inc, perform hedging on this basis, finally compare the results.

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2. Data

The data of this project is all from Yahoo Finance(https://ca.finance.yahoo.com), and the stock price data of Alphabet Inc. During the ten working days from August 22, 2022, to September 02, 2022. And the reference current stock price of Alphabet Inc is selected at 110.05 on September 09th, 2022.The stock price is shown below:

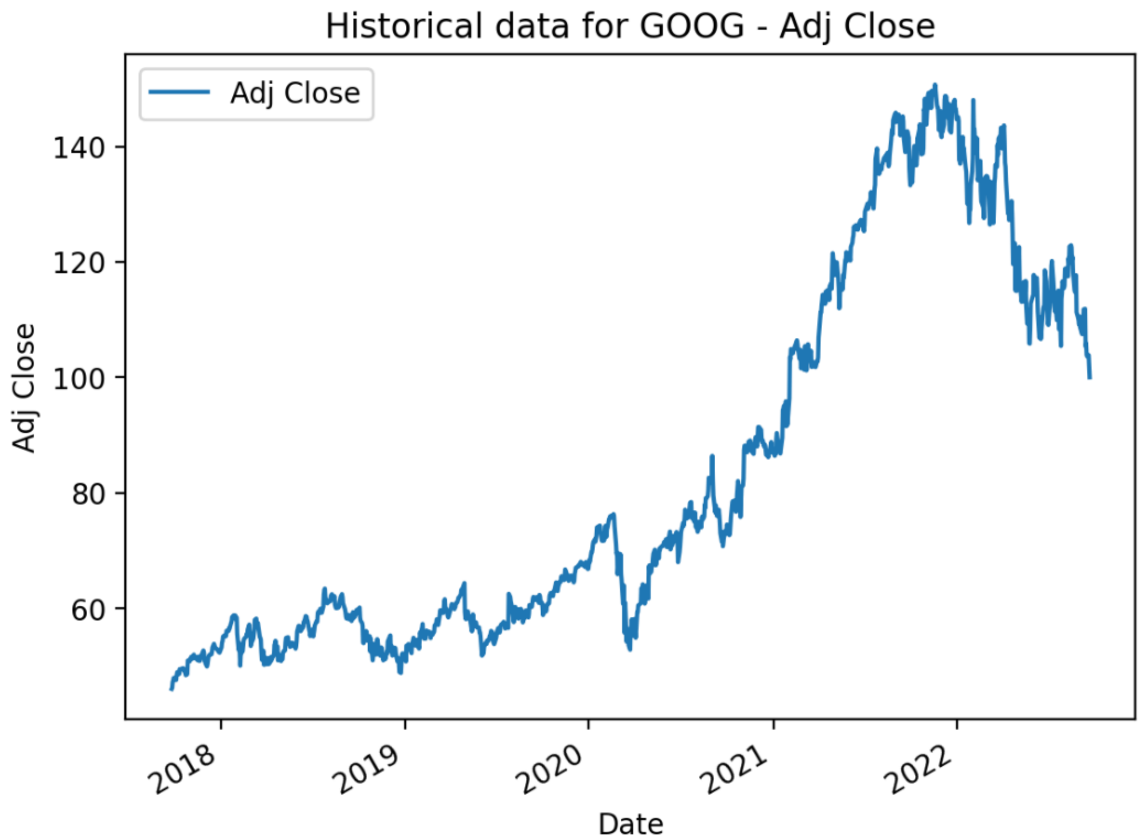

Figure 1: Price trend of Google from 2017 to 2022.

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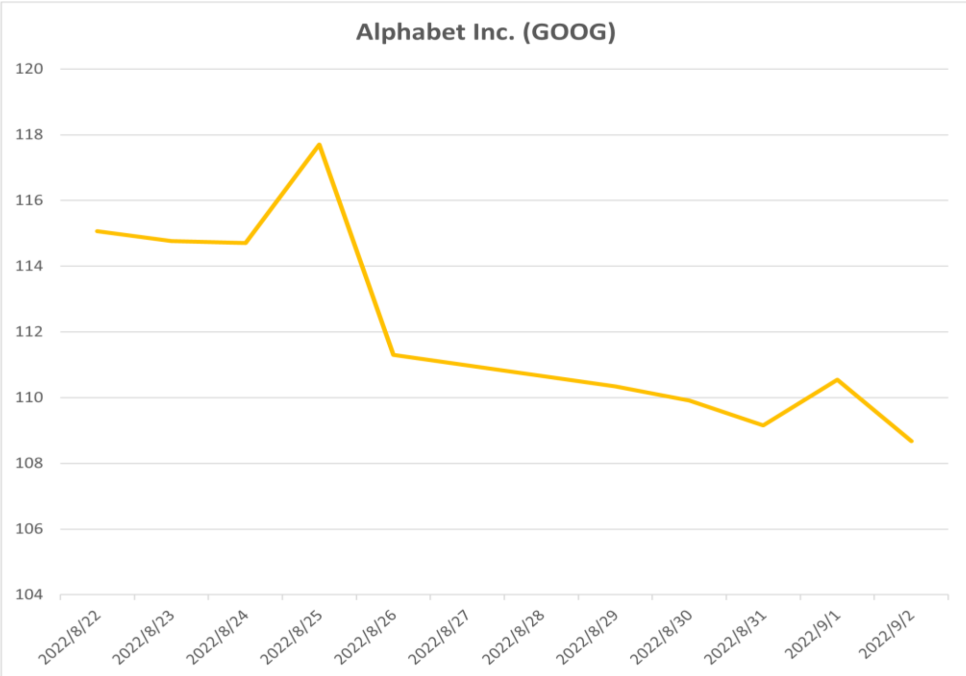

Figure 2: Price trend of Google from 08/22/2022 to 09/02/2022.

Ten historical data and ten options are applied throughout the calibration procedure. Alphabet Inc.'s changing adjusted closing prices from August 22, 2022, to September 02, 2022, were used as the benchmark stock prices for the historical data. A variety of data processing is done in advance, including adding the difference between the selected stock price and the price from the day before for the simulated implied volatility that will come later. As for ten options, considering the features that implied volatilities of the reference stocks are around the middle, and strike prices approximately equal to 105, five call options and five put options are collected, which share the same maturity on September 23, 2022, the hypothetical market price of each option is the midpoint of bid and ask. To be more accurate, the risk-free interest rate is referenced to the U.S. 3-Month T-Bill about 3.2%. In terms of details, the ten options contracts are shown in the table below.

Table 1: The 10 options selected for calibration.

Alphabet Inc.	Call options selected for calibration				
	GOOG220923C 00100000	GOOG220923C 00102000	GOOG220916C 00104000	GOOG220923C 00106000	GOOG220923C 00108000
	Put options selected for calibration				
	GOOG220923P 00102000	GOOG220923P 00104000	GOOG220923P 00106000	GOOG220923P 00108000	GOOG220923P 00110000

In the hedging process, a call option and a put option are chosen with the same maturity on September 30, 2022, to compare calibrated implied volatility to the actual market price when hedging. Additionally, strike prices that are close to the S0, which is 105, are maintained constant for the purpose of controlling variables. The following table displays the chosen option contracts.

Table 2: The 1 option selected for hedging.

Alphabet Inc.	Put option selected for hedging
	GOOG220930P00105000

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3. Methods

Use the data to build four distinct option pricing models after choosing the aforementioned information. The four models are broken down into the historical data model, BS model, Monte Carlo model, and BT model based on their various theoretical underpinnings. The four models all share nearly the same construction principles; the volatility, however, varies widely. While the BS, Monte Carlo, and BT models all employ implied volatility, the historical data model uses historical volatility.

One of the most crucial ideas in contemporary finance theory is the Black-Scholes model [4]. This formula is based on taking into account the theoretical value of investment instrument derivatives; additional risk factors that affect value, such as time, are also taken into account. It has been a crucial application tool in the history of options contract pricing ever since its creation in 1973. Another essential technique for valuing options is the binomial option pricing model [5], which was created in 1979. The usage iteration process, which permits arbitrary nodes or points in time between the valuation date and the option expiration date [6], forms the basis of this concept. In order to reduce the impact of errors on the final data, three distinct option pricing models are utilized in this study to complete the hedging of option A.

Note that the Google option we calculated is actually an American option, which we simply priced as a European option without considering the possibility that the option can be exercised early, and that the price of an American call option on a non-dividend-paying stock is the same as the price of a European call option. Theoretically, early exercise is not optimal when the stock does not pay a dividend. If the option will never be exercised early, then the price of the American option can be calculated like the European option.

Theoretically, risk neutral valuation is the foundation of Monte Carlo valuation. Here, the option's price reflects its discounted expected value; for more information, see rational pricing and risk neutrality. The method used is to first use simulation to produce a huge number of potential but random price routes for the underlying, and then to determine the associated exercise value (or "payoff") of the option for each path. Third, these payoffs are averaged, and finally, today's prices are also discounted. The value of the option is this outcome [7].

The option pricing model must be calibrated using data after being constructed, and all four models employ σ calibration. [8] The four model calibrations have notions that are largely consistent in the calibration model stage, but their performance tends to vary. The goal is to include the variable σ into one or more formulae that can reflect the magnitude of the erroneous value and to do so in accordance with the data that has been gathered to create a formula that is based on both the data and the value of σ. The magnitude of the error in the values at various data is the outcome of the reaction. After that, set σ as the independent variable, set the formula representing the error value and its result as the dependent variable, perform the calibration operation, and obtain the σ when the minimum error value is obtained, which is the ultimate goal of this step. Detailed model specifications are shown below.

In Historical Data Model, S stands for the stock price and k for the date, which ranges from August 22 to September 02. The second day based on t is thus represented as t+1. Get the stock prices' logarithm values for each date first. The difference in logarithm values between each pair of days is then determined using the equation. To determine the variance value of the equation, all of the results of the equation are employed (2). Assuming that there are 252 trading days per year, add it to equation (2) to obtain the final result of σ around 0.3562. This is because stock and option trading only take place on trading days. [9]

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$$Var = \frac{\sum((\ln(\frac{S_{t+1}}{S_t}) - \frac{(\sum \ln(\frac{S_{t+1}}{S_t}))}{T})^2}{T-1} \quad (1)$$

$$\sigma = \sqrt{Var \times 252} \quad (2)$$

A full formula for determining the price of an option or other financial derivative is the Black-Scholes model. The model generates a price for the option once all the financial inputs have been taken into account. Additionally, this enables traders to assess the effects of altering other formulaic parameters and assess the prospective effects on the option's price.

In BS Model, σ represents implied volatility, t represents time to maturity, S(t) represents the stock price, K represents the strike price of the option, and r represents the interest rate, which is assumed to be 3.2% in this model (3). First use equation (3) and equation (4) to calculate value of d_1 and d_2 which are the required elements for further calculation. Equation (5) and (6) calculate the call and put price of the option, and the $N(d_1)$ represents the normal distribution of d_1, $N(d_2)$ represents the normal distribution of d_2. After getting the values of the call and put options, put them into equation (7) to calculate the SSE (Residual sum of squares) error. C represents the call option prices, which calculate from equation (7), P represents the put option prices, which calculate from equation (8), and P_m represents the corresponding market values. At last, when the SSE value is the smallest, the desired σ value can be obtained.

$$d_1 = \frac{1}{\sigma\sqrt{t}} \ln\left(\frac{S_t}{K} + \left(r + \frac{\sigma^2}{2}\right)t\right) \quad (3)$$

$$d_2 = \frac{1}{\sigma\sqrt{(t)}} \ln\left(\frac{S_t}{K} + \left(r - \frac{\sigma^2}{2}\right)t\right) = d_1 - \sigma\sqrt{t} \quad (4)$$

$$C(t, S(t), K, r, \sigma) = S(t)N(d_1) - Ke^{-rt}N(d_2) \quad (5)$$

$$P(t, S(t), K, r, \sigma) = Ke^{-rt}N(-d_2) - S(t)N(-d_1) \quad (6)$$

$$SSE = \text{minimize}(\sigma) \sum_{i=1}^{n} \frac{(p_i(\sigma) - p_i^m(\sigma))^2}{p_i^m(\sigma)} \quad (7)$$

In terms of theory, Monte Carlo valuation relies on risk neutral valuation. And the same parameters as in the BS model are used.

$$dS_t = \mu S_t dt + \sigma S_t dZ_t \quad (8)$$

where dZ_t in (8) is the standard Wiener process (one-dimensional Brownian motion) whose increments are uncorrelated, and μ and σ are the annualized drift and variance rate of the underlying stock, respectively. Risk-neutral valuation justifies μ = r, and the solution of this differential equation is given by Ito’s equation, which yields a lognormally distributed random variable.[10]

Calculate the maturity index S_t(i) for all simulation results by using the maturity index formula above. And calculate the possible intrinsic value h_t(i) for each simulation of the expiration option.

$$h_t(\mathrm{i}) = \max(S_t(i) - K, 0) \quad (9)$$

$$P = e^{-rt} * \frac{1}{t} * \sum_i^I - h_t(\mathrm{i}) \quad (10)$$

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In the BT Model, u stands for the up coefficient in the binomial tree, d for the down coefficient, t for time to maturity, p for option pricing, and r for interest rate—which in this model is taken to be 3.2 percent. Using equations (11) and (12), get the up and down coefficients for each level of the binomial tree (12). The prices of the option are then obtained by adding each coefficient to equation (13). Equation (7) is used to determine the SSE error using the calculated option price and the related stock price. The desired value can be reached when the SSE value is the smallest.

$$u = e^{\sigma\sqrt{t}} \tag{11}$$

$$d = \frac{1}{u} \tag{12}$$

$$p = \frac{e^{rt} - d}{u - d} \tag{13}$$

The result of the aforementioned calculation can be utilized as the variable value once the model has been calibrated. The model can be coupled with Option A for hedging after it has been calibrated. The delta hedging approach is used by all four models [5]. In this stage, the historical data model, the BS model, and the BT model all use the same hedging strategy [11]. For the BS model, the portfolio value is represented by p(t), and each p(t) is obtained using equation (14), which is then used to complete the hedging process using equation (15). In the delta hedging phase, the BT model will also apply equation (16), where S stands for the option price. Collect the results of the four model hedges, compare the results, and draw a conclusion.

$$p(t) = p(t-1) + N(d_1)(S(t) - S(t-1)) \tag{14}$$

$$Loss\ with\ hedging(t) = S(t) - K - p(t) \tag{15}$$

$$Hedge\ ratio = \frac{D_u - D_d}{S_u - S_d} \tag{16}$$

4. Results

The four models had a practical impact on how the hedging effect manifested, according to the numerical results, and the historical data model has the best overall impact on how the results are presented.

Table 3: The gain/loss of holding one unit of the option on Alphabet Inc.'s stock with hedging by historical data model.

	Days	T-t	T_A	Stock	A(put)	Delta A(put)	Sell/Buy stock
2022/08/22	1	40	0.158730159	115.07	1.56	-0.18361	0.18361
2022/08/23	2	39	0.154761905	114.77	1.47	0.3	-0.11639
2022/08/24	3	38	0.150793651	114.7	1.45	0.285714	0.014286
2022/08/25	4	37	0.146825397	117.7	1.04	-0.13667	0.422384
2022/08/26	5	36	0.142857143	111.3	2.23	-0.18594	0.04927
2022/08/29	6	35	0.138888889	110.34	2.35	-0.125	-0.06094
2022/08/30	7	34	0.134920635	109.91	2.6	-0.5814	0.4564
2022/08/31	8	33	0.130952381	109.15	2.5	0.131579	-0.71298
2022/09/01	9	32	0.126984127	110.55	2.13	-0.26429	0.395869
2022/09/02	10	31	0.123015873	108.68	2.69	-0.29947	0.03518

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According to the table above, the result of hedging using the historical data model shows that the final loss is $0.5215 per unit.

Table 4: The gain/loss of holding one unit of the option on Alphabet Inc.'s stock with hedging by BS model.

	Days	T-t	T_A	Stock	A(put)	Delta A(put)	Sell/Buy stock
2022/08/22	1	40	0.158730159	115.07	1.4958	-0.12705	0.12705
2022/08/23	2	39	0.154761905	114.77	1.5096	-0.046	0.17305
2022/08/24	3	38	0.150793651	114.7	1.4784	0.445714	-0.49171
2022/08/25	4	37	0.146825397	117.7	0.9472	-0.17707	0.622784
2022/08/26	5	36	0.142857143	111.3	2.1634	-0.19003	0.01296
2022/08/29	6	35	0.138888889	110.34	2.3799	-0.22552	0.03549
2022/08/30	7	34	0.134920635	109.91	2.4522	-0.16814	-0.05738
2022/08/31	8	33	0.130952381	109.15	2.6341	-0.23934	0.0712
2022/09/01	9	32	0.126984127	110.55	2.1476	-0.3475	0.10816
2022/09/02	10	31	0.123015873	108.68	2.6686	-0.27861	-0.06889

According to the table above, the result of hedging using the BS model shows that the final loss is $0.522932 per unit.

Table 5: The gain/loss of holding one unit of the option on Alphabet Inc.'s stock with hedging by BT model.

	Days	T-t	T_A	Stock	A(put)	Delta A(put)	Sell/Buy stock
2022/08/22	1	40	0.158730159	115.07	1.4732	-0.12646	0.12646
2022/08/23	2	39	0.154761905	114.77	1.4855	-0.041	0.16746
2022/08/24	3	38	0.150793651	114.7	1.4553	0.431429	-0.47243
2022/08/25	4	37	0.146825397	117.7	0.9271	-0.17607	0.607499
2022/08/26	5	36	0.142857143	111.3	2.1376	-0.18914	0.01307
2022/08/29	6	35	0.138888889	110.34	2.3562	-0.22771	0.03857
2022/08/30	7	34	0.134920635	109.91	2.4287	-0.1686	-0.05911
2022/08/31	8	33	0.130952381	109.15	2.6051	-0.23211	0.06351
2022/09/01	9	32	0.126984127	110.55	2.1196	-0.34679	0.11468
2022/09/02	10	31	0.123015873	108.68	2.6449	-0.28091	-0.06588

According to the table above, the result of hedging using the BT model shows that the final loss is $0.565368 per unit.

Table 6: The gain/loss of holding one unit of the option on Alphabet Inc.'s stock with hedging by Monte Carlo model.

	Days	T-t	T_A	Stock	A(put)	Delta A(put)	Sell/Buy stock
2022/08/22	1	40	0.11111111	115.07	1.494	-0.13337	0.13337
2022/08/23	2	39	0.10833333	114.77	1.5085	-0.04833	0.1817
2022/08/24	3	38	0.10555555	114.7	1.4796	0.41285	-0.46118
2022/08/25	4	37	0.10277777	117.7	0.9493	-0.17676	0.58961
2022/08/26	5	36	0.1	111.3	2.1651	-0.18996	0.0132

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Table 6: (continued).

2022/08/29	6	35	0.09722222	110.34	2.3772	-0.22093	0.03097
2022/08/30	7	34	0.09444444	109.91	2.4514	-0.17255	-0.04838
2022/08/31	8	33	0.09166666	109.15	2.6345	-0.24092	0.06837
2022/09/01	9	32	0.08888888	110.55	2.1479	-0.34757	0.10665
2022/09/02	10	31	0.08611111	108.68	2.6687	-0.27850	-0.06907

According to the table above, the result of hedging using the Monte Carlo model shows that the final loss is $0.522938 per unit.

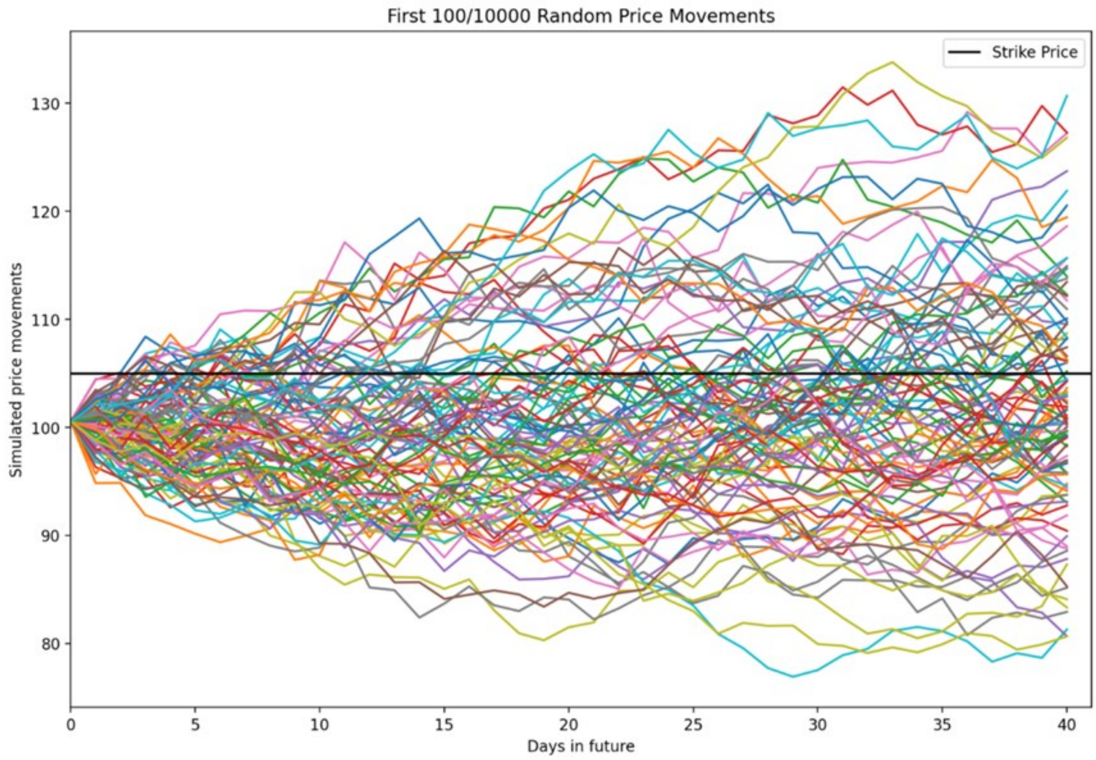

Figure 3: Simulation in Monte Carlo model.

5. Discussion

The findings section demonstrates that the Monte Carlo approach with the BS model hedging strategy and the historical data-based option pricing model both produce strong outcomes, whereas the BT model-based hedging strategy does not. In terms of the evaluation method of delta, the historical data model is unquestionably more accurate. The stock price and option data used to determine the parameter values for this model are actual. Monte Carlo methods can be used to analyze the value of an option with various sources of uncertainty. First, a variety of pathways or trajectories are used to generate random numbers in order to replicate the price of the underlying asset. The estimated price of options can be determined by repeatedly modeling the trajectories and obtaining suitable averages; this estimate is compatible with the analytical outcomes from the Black-Scholes model. The BS model makes the assumption that some factors in its calculation will remain constant. These elements—volatility and risk-free rate of return—unfortunately fluctuate constantly. The calibrated values $\boldsymbol{\sigma}$ are different from those of the historical data model and the BS model when the BT model is incorporated into the option pricing model. In addition, compared to the previous three models, the BT model's hedging procedure involves a small number of additional computational processes. The likelihood of errors rising as the number of calculation steps rises. A thorough investigation of the BT model reveals that the final hedging outcome is unsatisfactory due to the bigger mistake in the value $\boldsymbol{\sigma}$, and the larger error introduced by the increased hedging stages.

6. Conclusion

An asset with payoffs that depend on the value of an underlying asset is called an option. A put option gives its holder the right to sell the underlying asset at a fixed price at any time before the option expires, whereas a call option gives its holder the right to acquire the asset at a given price. The current value of the underlying asset, its volatility, the strike price, the duration of the option, the riskless interest rate, and the anticipated dividends on the asset are the six factors that define an option's value. The Black-Scholes, Binomial, and Monte Carlo models, which value options by building replicating portfolios made up of the underlying asset and riskless lending or borrowing, all serve as examples of this. These models can be used to value assets with characteristics similar to options. The stock option pricing models for a single business were created using several techniques in this article, and the consequences of the hedging were illustrated. Likewise, consider and debate how various hedging pricing models have performed. The historical data model performs best when comparing the results in terms of hedging outcomes and has the least amount of unit loss. This study highlights how different option pricing models influence how different hedging techniques behave when covering stock options of the same business. This allows investors' flexibility in choosing multiple hedging strategies and option pricing models for their portfolio investments.

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