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On Frontier Portfolio in Shanghai Stock Exchange Based on Mean Variance Model

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Abstract: In security markets, how to achieve optimal allocation of asset has become the focus of investors. The mean-variance model is a method to achieve revenue maximization and risk minimization on investing stock portfolio. This paper will research on investment problem in Shanghai Stock Exchange. It involves research result about mean variance model since 1952. To take the experiment, 10 stocks are chosen from Shanghai Stock Exchange. Organizing rates of return for 360 days and using the average daily rate of return as the expected rate of return. Applying mean variance model, selecting data, the curve of frontier portfolio is obtained by experiment. Based on the result, the investment advice is given. It shows that higher expected rate of return accompanies by higher risk. To achieve higher returns, investor could analyse the variation of each asset along with the rise of the expected rate of change. When negative weight appears, there exists short selling. If short selling is not allowed, investor could shrink the interval of the expected rate of return.

Keywords: mean variance model, frontier portfolio, investment portfolio

1. Introduction

In security markets, how to achieve optimal allocation of asset has become the focus of investors. Markowitz put forward mean–variance (MV) model to quantify the revenue and the risk [1]. The MV model is a crucial tool nowadays for maximising revenue while minimizing risk. But the complexity of computing the covariance matrix makes it less useful.

This paper will test mean–variance model by example and find out the portfolio frontier, for better investment decision. Sun [2] note that the number of assets of asset portfolio is one of the aspects investors considered. Portfolio diversification will reduce the risk; however, it adds the cost of portfolio management. Therefore, there exists an investment proportion to achieve minimum risk of portfolio. Considering the vagueness in real life, Zhou et al. [3] apply fuzzy optimization to the model which focus on income, risk and skewness. Under the vagueness, Jana et al. [4] put forward a multi-objective optimization model which considered transaction cost. With the development of computer science, genetic algorithm is a good method for analysing practical examples. However, the operation process may lose stabilization. It is important to establish suitable models to find the optimum portfolio.

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The following structure of this paper will contain a short literature review of previous works. Section 3 and 4 will give example and use mean variance model to solve portfolio frontier. Section 5 will analyse the curve of portfolio frontier and give advice to investors. Section 6 is a short conclusion.

2. Review

The original MV model took into consideration expected return, portfolio weight, and asset covariance. When investment return confirms, the model is aiming to find the least risky portfolio. When risk determines, the model is aiming to find the maximal expectation profit. It forms the framework of the following study.

Targeting at the diversity of the reality market, for example, whether to allow short sale, MV model have been extended. Ma and Tang have put out the discussion under two cases, no short sale and restricted short sale [5-6]. When short sale is not allowed, the investment proportion vector of original MV model should be constrained. When short sale is allowed and restricted, the short-selling of a security could be considered as investing in another security. The author found that the short-squeezed impacts short sale which allows pledge more than which requires earnest money. The discussion provides methods applied in different markets.

The mean-variance portfolio optimization in continuous time is the subject of Bjork's study [7]. A game theoretic framework is used to tackle the issue, and subgame perfect Nash equilibrium solutions are sought after. It is possible to define the idea of a "subgame perfect Nash equilibrium point" for the game. The formal definition of an equilibrium control could be provided based on the control law. The concept of an equilibrium and an optimal strategy will be the same in a typical time-consistent environment, and the equilibrium value function will be the typical optimal value function. They arrive to the result that the share of wealth invested in the stock declines as the horizon lengthens in the mean variance problem with state-dependent risk aversion by altering the parameters.

Inflation exists chronically, it has become an important factor for investors to consider. Yao et al. study on continuous-time mean-variance portfolio selection under inflation [8]. Using Lagrange multiplier technique, the MV model could be a standard stochastic optimal control problem. Using the dynamic programming methodology, the efficient investment strategies and mean-variance efficient frontier are then expressed in closed form. Five stocks from Shanghai and Shenzhen Stock Exchange and the data from 2010/5/4 to 2011/6/1 are used to show the correctness of the model. The study shows that when considering inflation, efficient frontier is a branch of hyperbola, instead of a straight line on the standard deviation-mean plane.

To achieve the most efficient allocation of risk in risk portfolio management, Jiang et al. try to find the commonality between the risk sharing problem and the risk budgeting problem [9]. To increase the predicted utility level of the group, the Pareto optimal income distribution ratio is sought after. The Pareto efficient risk sharing ratio could be deduced by the first order optimal condition, then the Pareto efficient risk sharing principle of risk averse individuals under mean-variance expected utility function could be obtained. It draws a conclusion that when the formation conditions of risk sharing groups are considered, the Pareto efficient risk sharing principle corresponding to risk sharing is equivalent to the conditional expected risk allocation function.

Investors should focus on the number of assets in the asset portfolio, it is easier to manage fewer assets when returns and risks are fixed. Sun [2] improved MV model basing on multi-objective optimization. Using binary variables, the additional objective of minimizing the number of assets in the asset portfolio is added. The variable is 1 when the asset is held, then otherwise 0. To test the feasibility of the model, the author selected four stock portfolios differ from amounts, and using NSGA-II algorithm. The model proved that diversification of investment could reduce risks, but beyond a certain amount would increase the cost of asset management.

Brodie et al. notice that there would make great effects due to small changes, volatilities, or correlations in MV model. A penalty proportional to the sum of the absolute values of the portfolio weights is added to the objective function [10]. By using 48 industry portfolios and 100 portfolios and data from 1971 to 2006, the conclusion is that the optimal sparse portfolios outperform the evenly weighted portfolios by achieving a smaller variance; moreover, they do so with only a small number of active positions, and the effect is observed over a range of values for this number.

The risk is the difference between the expected rate of return and the real rate of return, which caused by uncertainty. Chen simplify the quadratic constraint in MV model into fuzzy linear constraint by using fuzzy constraint. According to the uncertainty of investor's returns on risk assets [11], a fuzzy portfolio optimization model was built and transformed into a multi-objective linear programming model. Based on the data set by Chen, Deng discusses the portfolio model which rate of return satisfies the trapezoidal fuzzy number distribution [12]. A double objective MV model was established to achieve maximal returns and minimal risks. It proves that increasing the number of portfolio objective functions and limiting conditions would smooth the effective boundary within the range of risks or returns investors accept. The risks and returns of the optimal portfolio would both reduce, but the future returns would be more stable.

Realistic factors are introduced to extend the dynamic MV model. However, investors do not know the parameters precisely and estimate. The estimation deviation would bring estimation risk. Li and Yuan try to introduce the parameter uncertainty or estimated risk into portfolio selection model and analyse the influence [13]. The information set investors observe could be considered as the domain flow generated by the stock price process. Investors tend to choose the investment strategy with higher terminal wealth expectation and lower variance. Martingale method is applied to divide the problem into two subproblems, then find the optimal terminal wealth and solve the dynamic investment respectively. By testing the parameters obtained from the estimation based on the data of China's securities market, it shows that after introducing the parametric uncertainty, the optimal wealth proportion invested in stocks at the initial moment is smaller than the case where parametric uncertainty is ignored. It also shows that parametric uncertainty is more important to investors with long period of investment plan.

3. Methodology

Assume that there are N≥2 risky assets sold in a market with no friction, unlimited short selling, and finite variation and uneven expectations for the rates of return on these assets.

Let w_p denotes the N-vector portfolio weights of p. If w_p is the solution to the quadratic program:

$$\min_{\{w\}} \frac{1}{2} w^T V w \tag{1}$$

s.t.

$$w^T e = E[\tilde{r}_p] \tag{2}$$

and

$$w^T 1 = 1 \tag{3}$$

the portfolio p is a frontier portfolio.

Where V denotes variance -covariance matrix of the rates of return of assets, e denotes the N-vector of expected rates of return on the N assets, $E[\tilde{r}_p]$ denotes the expected rate of return on portfolio p, and 1 is an N-vector of ones.

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w_p is the solution to:

$$\min_{\{w,\lambda,\gamma\}} L = \frac{1}{2} w^T V w + \lambda(E[\tilde{r}_p] - w^T e) + \gamma(1 - w^T 1) \tag{4}$$

λ and γ gives

$$\lambda = \frac{CE[\tilde{r}_p] - A}{D} \tag{5}$$

$$\gamma = \frac{B - AE[\tilde{r}_p]}{D} \tag{6}$$

where

$$A = 1^T V^{-1} e = e^T V^{-1} 1 \tag{7}$$

$$B = e^T V^{-1} e \tag{8}$$

$$C = 1^T V^{-1} 1 \tag{9}$$

$$D = BC - A^2$$

The unique set of portfolio weights for the frontier portfolio having an expected rate of return of $E[\tilde{r}_p]$:

$$w_p = g + hE[\tilde{r}_p] \tag{10}$$

where

$$g = \frac{1}{D}[B(V^{-1}1) - A(V^{-1}e)] \tag{11}$$

$$h = \frac{1}{D}[C(V^{-1}e) - A(V^{-1}1) \tag{12}$$

In the $\sigma(\tilde{r}) - E[\tilde{r}]$ space, portfolio frontier:

$$\sigma^2(\tilde{r}_p) = \frac{1}{D}(C(E[\tilde{r}_p]^2 - 2AE[\tilde{r}_p] + B) \tag{13}$$

the minimum variance portfolio is at $(\sqrt{\frac{1}{C}}, \frac{A}{C})$.

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4. Result

Choosing 10 portfolios from SSE.

Table 1: 10 portfolios.

1	600019
2	600006
3	600007
4	600008
5	600018
6	600039
7	600099
8	600106
9	600000
10	600166

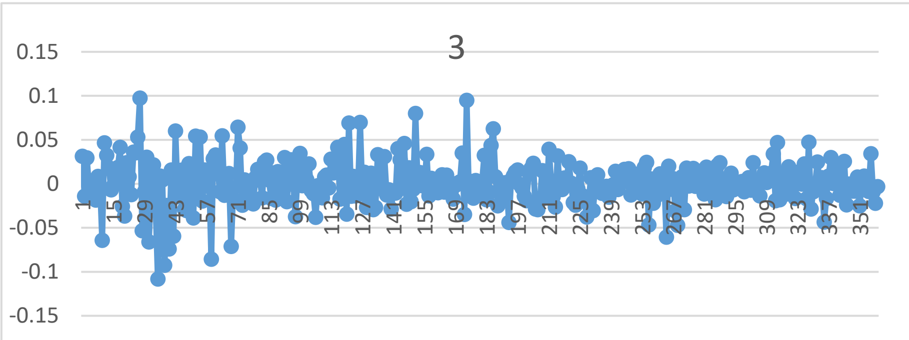

Figure 1: Daily rate of return of portfolio 3.

Organizing rates of return for 360 days ended in 2017-5-9. Line charts of daily rate of return of some portfolio(Fig.1-2)

Using the average daily rate of return as the expected rate of return, respectively(table2).

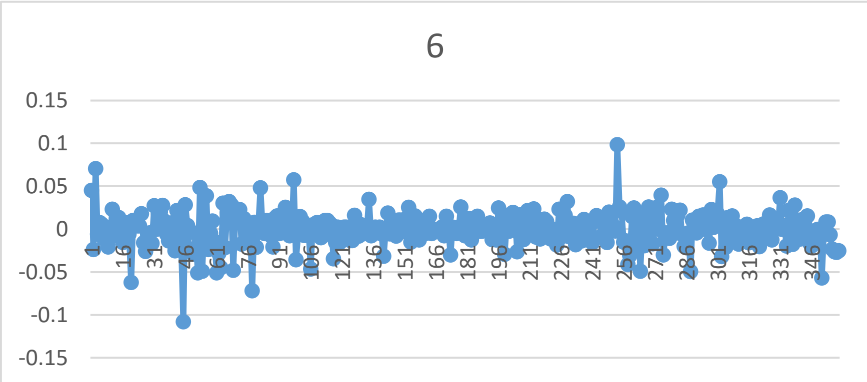

Figure 2: Daily rate of return of portfolio 6.

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Table 2: Expected rate of return of 10 assets.

e	
1	-0.00035%
2	-0.00080%
3	0.00095%
4	-0.00003%
5	-0.00048%
6	-0.00017%
7	0.00179%
8	-0.00099%
9	-0.00010%
10	-0.00162%

Table 3: Variance -covariance matrix of the rates of return.

V(E-05)	1	2	3	4	5	6	7	8	9	10
1	86.64	10.21	6.02	-2.79	-1.84	3.85	-4.14	3.47	-0.76	1.26
2	10.21	87.91	4.73	-0.39	1.09	6.00	6.78	-0.46	-3.08	3.57
3	6.02	4.73	64.59	-8.75	17.72	3.62	4.92	1.37	2.04	2.32
4	-2.79	-0.39	-8.75	159.71	0.06	18.89	-5.82	1.46	0.72	-5.83
5	-1.84	1.09	17.72	0.06	31.44	2.34	4.51	7.98	3.29	-0.70
6	3.85	6.00	3.62	18.89	2.34	40.09	-2.67	5.16	4.49	4.20
7	-4.14	6.78	4.92	-5.82	4.51	-2.67	117.02	4.84	7.56	3.51
8	3.47	-0.46	1.37	1.46	7.98	5.16	4.84	182.88	8.61	-5.83
9	-0.76	-3.08	2.04	0.72	3.29	4.49	7.56	8.61	19.77	-0.63
10	1.26	3.57	2.32	-5.83	-0.70	4.20	3.51	-5.83	-0.63	111.37

The variance -covariance matrix of the rates of return on risky assets(table 3)
A, B, C, D and $\frac{A}{C}$ (E-06) (table 4)

Table 4: A, B, C, D and $\frac{A}{C}$.

A	-31056.391
B	1.08099
C	12425405470
D	12467.227
$\frac{A}{C}$	-2.49943

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Table 5: g and h.

g	h
0.081482417	-1306.929243
0.055961054	-8324.13931
0.107308733	24842.03632
0.050874544	2411.378876
0.122749166	-23416.52566
0.111330854	4521.334395
0.084011362	18405.9598
0.005491444	-4278.488089
0.345255977	623.9584715
0.035534449	-13478.58556

g and h(table 5)

On portfolio, setting the expected rate of return from -0.0005 to 0.0005(table 6)

Table 6: Expected rate of return, weight vector of assets and standard deviation.

Expected rate of return (E-06)	5	4	3	2	1	0	-1	-2	-3	-4	-5
Weight vector of 10 assets (%)	7.49	7.63	7.76	7.89	8.02	8.15	8.28	8.41	8.54	8.67	8.80
	1.43	2.27	3.10	3.93	4.76	5.60	6.43	7.26	8.09	8.93	9.76
	23.15	20.67	18.18	15.70	13.22	10.73	8.25	5.76	3.28	0.79	-1.69
	6.29	6.05	5.81	5.57	5.33	5.09	4.85	4.61	4.36	4.12	3.88
	0.57	2.91	5.25	7.59	9.93	12.27	14.62	16.96	19.30	21.64	23.98
	13.39	12.94	12.49	12.04	11.59	11.13	10.68	10.23	9.78	9.32	8.87
	17.60	15.76	13.92	12.08	10.24	8.40	6.56	4.72	2.88	1.04	-0.80
	-1.59	-1.16	-0.73	-0.31	0.12	0.55	0.98	1.40	1.83	2.26	2.69
	34.84	34.78	34.71	34.65	34.59	34.53	34.46	34.40	34.34	34.28	34.21
	-3.19	-1.84	-0.49	0.86	2.21	3.55	4.90	6.25	7.60	8.94	10.29
Standard deviation	0.011684734	0.011071636	0.010517724	0.010032806	0.009627314	0.00931163	0.009095109	0.008984924	0.008984988	0.009095298	0.009311937

In the $\sigma(\tilde{r}) - E[\tilde{r}]$ plane, portfolio frontier(Fig. 3)

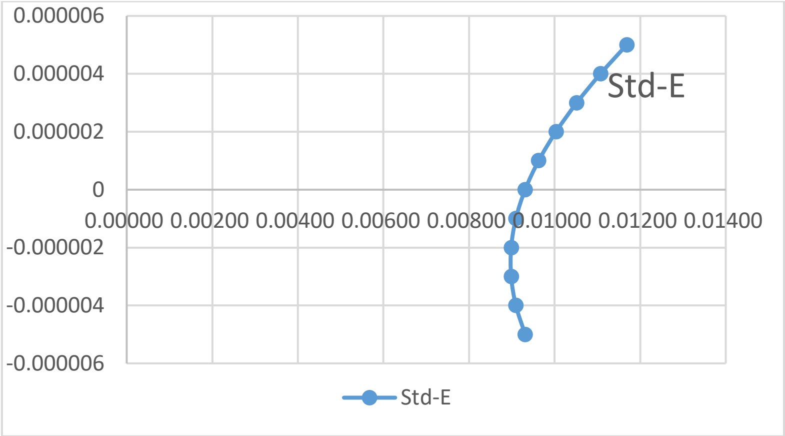

Figure 3: Portfolio frontier.

5. Discussion

The curve of portfolio frontier is one side of the hyperbola. It shows that higher expected rate of return accompanies by higher risk.

To achieve higher returns, investor could analyse the variation of each asset along with the rise of the expected rate of change. If the asset was increasing with the rise of the expected rate of return, it should be purchased more. Conversely, the asset which decreasing with the rise of the expected rate of return should be purchased less. For example, in portfolio above, asset 3 should be purchased more and asset 5 should be less.

When negative weight appears, there exists short selling. In portfolio above, only when the expected rate of return between -0.000004 and 0.000001 would not appear short selling. Investor should notice the restricted condition of short selling. If short selling is not allowed, investor could shrink the interval of the expected rate of return.

6. Conclusion

In this paper mean-variance model is discussed. To achieve revenue maximization and risk minimization, it is applied to solve the problem of achieving optimal allocation of asset.

Based on different conditions, previous researches involve short sale, continuous time, inflation, risk sharing, multi-objective optimization, penalty, fuzzy constraint and martingale method.

This paper selects 10 portfolios from SSE and applies mean-variance model to find optimal allocation. Setting the average daily rate of return of each portfolio as the expected rate of return, obtaining covariance matrix. After operation, solving out the curve of portfolio frontier.

The curve of portfolio frontier shows that higher expected rate of return accompanies by higher risk. If one asset was increasing with the rise of the expected rate of return, it should be purchased more, vice versa. When negative weight appears, there exists short selling. If short selling is not allowed, investor could shrink the interval of the expected rate of return.

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