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Volatility, Uncertainty, and Option Pricing

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Abstract: Amidst global and Chinese uncertainties, this paper delves into stock market volatility and option pricing within the current economic policy context. Focusing on China's stock market, it calculates returns and volatility changes in key indices, analyzing differences among indices and funds, along with temporal variations. Shared data characteristics and reflections on new market trends emerge. Employing methods such as the GARCH model, Black-Scholes formula, and Monte Carlo algorithm, it analyzes volatility and option pricing using extensive annual and monthly data. Visualizations showcase market change patterns. The study defines volatility as a measure of financial asset price fluctuation extent, reflecting asset risk. Higher volatility indicates pronounced price fluctuations and uncertainty, while lower volatility signifies smoother fluctuations and greater certainty. Merging data with volatility's significance, the study probes China's securities market uncertainty, investigating the link between option pricing and volatility. It concludes by identifying the connection between volatility, uncertainty, and option pricing, pointing to future research directions and challenges. Future work will track the latest market trends to enrich understanding.

Keywords: volatility, GARCH model, Monte Carlo algorithm, Black-Scholes pricing formula, uncertainty

1. Introduction

In today's world, uncertainty has become a prevailing norm. From an international perspective, factors such as the ongoing aftermath of the pandemic, potential risks arising from geopolitical crises, and global economic growth deceleration pose significant challenges to the financial industry. The Economic Policy Uncertainty (EPU) index serves as a metric to gauge economic policy uncertainty. This index reflects the uncertainty that global investors perceive in economic policies, encompassing political, economic, and social aspects. The changing trend of China's EPU over the past two decades is depicted in the following figure:

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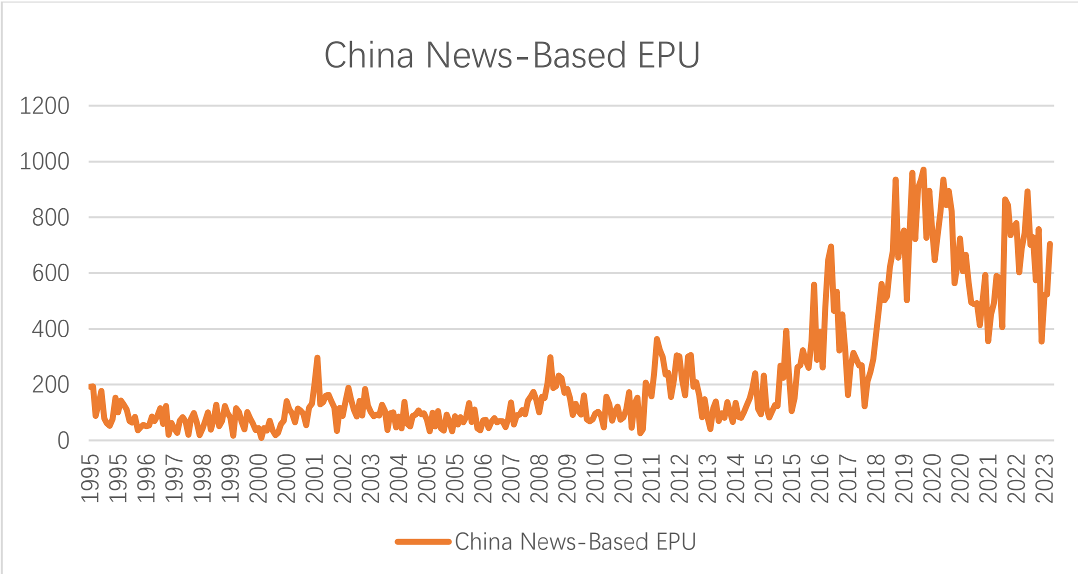

Note: Data source http://www.policyuncertainty.com/

Figure 1: Changing trend of China's EPU.

In recent years, the EPU index has consistently remained at a higher level, indicating the escalating uncertainty in China's economic domain. Political uncertainties hold far-reaching impacts on the global economy and financial markets. For instance, events like the US presidential elections and the Russia-Ukraine conflict could alter the global economic landscape, affecting exchange rates, trade policies, economic growth prospects, etc. Economic uncertainties also significantly influence global financial markets. Economic recessions in emerging markets, escalating international trade barriers, and employment concerns arising from technological shifts contribute to uncertainty in the global financial markets.

For China, internal and external uncertainties also impact the financial industry. Matters such as domestic macroeconomic stability, financial regulatory policies, and corporate debt could influence the financial markets. Global economic growth slowdowns and trade protectionism contribute to volatility in China's financial markets.

These uncertainties primarily manifest their impact on the financial industry and financial derivatives in the following ways: First, they can exacerbate market volatility and amplify panic sentiments, leading to substantial asset price declines, market liquidity tightness, and other risks. Second, they can trigger credit risks within financial institutions, resulting in debt defaults and asset impairments. Third, they can influence the financial derivatives market, causing decreased trading volumes and reduced risk preferences among market participants. In the current globally uncertain environment, the financial industry and financial derivatives market face both challenges and opportunities. Maximizing investor returns necessitates a thorough understanding and management of various uncertainty factors.

Options, as one of the financial derivatives, hold a significant position in the financial market. The uncertainty of options is influenced by market conditions and environments, closely tied to option pricing. Volatility, as a measure of uncertainty, plays a pivotal role in option pricing. The development of China's options market can be summarized in four aspects:

Initial stage of development: China's options market was in its nascent stage of development. Prior to 2015, the market was predominantly dominated by over-the-counter options, while on-exchange options were introduced by the Shanghai Stock Exchange only in 2015. In terms of off-exchange

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options, the trading volumes in both the interbank market and exchange market had reached significant levels, although on-exchange options market was relatively smaller.

Rapid development: Despite a later start, China's options market has experienced rapid growth. Taking the Shanghai 50ETF options as an example, its trading volume has surpassed that of many mature market options, establishing itself as a vital risk management tool in the Chinese capital market.

Expansion of option contract types and scales: With the continuous development of the options market, the types and scales of options have expanded. Presently, China has launched various types of options including equity options, commodity options, and financial futures options. Simultaneously, exchanges are actively pushing for the introduction of new option varieties.

Diversified options trading methods: In addition to on-exchange trading at securities companies, investors can also access option investment opportunities through private transactions with others.

In summary, China’s options market, while still in its initial developmental stage, has demonstrated a trajectory of rapid growth, and it holds the potential to become a significant component of the capital market in the future. Amid uncertainties, comprehensive research on options and option pricing aids in understanding and assessing risks in financial markets, providing effective risk management tools for investors. As a financial derivative, options help investors mitigate portfolio risks and enhance returns. Research on option pricing contributes to the reasonable determination of market prices, upholds market stability, and promotes the healthy development of financial markets.

2. Literature Review

One of the key issues in option pricing is the valuation of assets under conditions of uncertainty. Uncertainty is a characteristic of financial markets, and traditional finance theory assumes rational investors in asset pricing under uncertain circumstances. This has given rise to models such as the Capital Asset Pricing Model, the Three-Factor Model, and the Black-Scholes option pricing model [1]. In this context, uncertainty refers to the probabilistic uncertainty of future events, i.e., traditional risk, leading to the existence of a definite equilibrium price for assets. Uncertainty is omnipresent, impacting underlying asset prices sometimes stably and other times unpredictably. There are different types of asset uncertainties. Knight distinguished between known uncertainty (risk) and unknown uncertainty. The former is confined to a unique probability distribution that is known, while in the latter case, some probability measure information is known, but not all required information can be accurately or fully ascertained. The type of distribution is known, but the distribution parameters are not [2].

For complex and variable financial markets, the volatility of underlying asset prices is just one aspect affecting option prices, and option pricing is one of the pivotal issues in finance. The classical option pricing formula includes the Black-Scholes pricing formula. The Black-Scholes option model's assumptions are overly idealized, as its assumptions of log-normal distribution of returns, continuous trading processes, and constant volatility are often hard to meet in reality [3]. Research on financial asset return and derivative price time series has shown at least three deviations from Brownian motion. Firstly, asset price jumps lead to non-normal distribution of returns. Secondly, asset return volatility varies stochastically over time. Thirdly, there exists a "leverage effect," manifested by a negative correlation between asset returns and their volatility, which is typically the case for equity-based financial assets. Furthermore, research has found systematic biases in the Black-Scholes model for pricing stock index options, particularly when options are out of the money. Additionally, the joint estimation of stock returns and volatility derivatives using the GARCH option pricing approach has proven highly beneficial, highlighting the utility of volatility derivatives in GARCH option valuation [4].

In the context of China's options market, scholars have been exploring the constant-setting issues of volatility and interest rates in option pricing since the introduction of the Black-Scholes model. Introducing floating interest rates into local volatility models has improved pricing effectiveness. Utilizing data from the Shanghai and Shenzhen 300 stock index options, a comparison was made between in-sample pricing errors and out-of-sample pricing errors, with empirical results suggesting the superiority of the stochastic volatility incentive model over the stochastic volatility incentive model [5]. The option pricing problem under Brownian motion is controlled by the fractional diffusion equation of controlled subordinate Brownian motion, characterized by self-similarity, peakedness, long memory, and other financial properties. These properties suggest that fractional calculus can aptly describe financial data [6]. Researching option pricing problems under the assumption that stock prices are driven by subordinate Brownian motion can derive closed-form pricing formulas for European options.

Options are widely traded in China's stock, fixed-income fund, foreign exchange, and commodities markets, playing significant roles in speculative profits and risk hedging. This has sparked interest among researchers in option valuation. Existing option pricing methods can be broadly categorized into numerical analysis methods and analytical approximation methods. Numerical methods include binomial tree methods and Monte Carlo methods; however, they are time-consuming and may sacrifice accuracy due to computational costs [7]. Hence, research on analytical approximation methods has become an important field. Classic Black-Scholes option pricing models play a vital role in financial markets, and investigating option pricing problems is a key factor and pioneer in modern financial engineering and computational fields, yielding fruitful results in practical economic life [8]. In numerous hidden options such as insurance premium pricing and intangible assets, valuation, prepaid term deposit, various convertible options, the prices of financial instruments imply volatility and uncertainty. The theory and models of option pricing are highly complex, yet their utilization is simple. Financial derivatives, including stocks, bonds, currencies, commodities, and more, are reasonably valued. The Black-Scholes option pricing model assumes that asset prices change continuously and follow Brownian motion. Research indicates that asset prices sometimes change discontinuously, experiencing jumps in unforeseen circumstances, which can lead to prediction biases in option pricing [9].

In this paper, the classical Black-Scholes pricing formula is used to compute option prices and Greek letters. The Monte Carlo method is adopted for option pricing, and the GARCH model is used to analyze the monthly volatility of the Shanghai and Shenzhen indices. Additionally, both the Monte Carlo and Black-Scholes pricing formula methods are employed to analyze the volatility and option pricing of multiple stock indices, investigating the uncertainty of stock market options. Using intuitive line graphs, the monthly and annual changes in volatility are reflected, confirming that volatility is not entirely constant as assumed by the ideal model, but rather significantly relates to the year and bull-bear market cycles. Different indices and funds exhibit varying patterns of volatility changes. Consequently, the value of target options will be influenced by instantaneous volatility under other equivalent conditions. Pricing options using the Black-Scholes formula achieves good results under relatively stable volatility conditions.

3. Data and Models

3.1. Black-Scholes Pricing Formula

Let V (0, S0) represent the fair price (at time 0) of a European call option with a strike price of K and expiration date T. The Black-Scholes option valuation formula is given by:

$$V(t, S_0) = S_0\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2) \tag{1}$$

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Short Hedge for European Call Options:
A holding of $\Delta = c_x(t,x)$ shares of stock is needed, with a value of $xc_x(t,x)$.
The cash market account position is:

$$M = c(t,x) - xc_x(t,x) = -Ke^{-r(T-t)}\Phi(d_2) \tag{2}$$

Based on the BS formula:

$$c(t,x) = x\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2) = xc_x(t,x) - Ke^{-r(T-t)}\Phi(d_2) \tag{3}$$

Long Hedge for European Call Options:
A short sale of $\Delta = c_x(t,x)$ shares of stock is required, with a value of $xc_x(t,x)$.
The cash market account position is:

$$M = xc_x(t,x) - c(t,x) = Ke^{-r(T-t)}\Phi(d_2) \tag{4}$$

3.2. Brownian Motion

Geometric Brownian Motion (GBM), also known as exponential Brownian motion, is a continuous-time stochastic process where the logarithm of the random variable follows a Brownian motion process. In the case where the stochastic differential equation is satisfied, the process St is considered to follow geometric Brownian motion:

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t) \tag{5}$$

Here, Wt is a Wiener process, or Brownian motion, and the drift percentage μ and volatility percentage σ are constants.

3.3. Monte Carlo Method

The Monte Carlo method, or Monte Carlo experiment, is a class of computational algorithms that relies on repeated random sampling to obtain numerical results. The fundamental concept is to use randomness to solve problems that may, in principle, have deterministic solutions. Law of Large Numbers: If $X_1, X_2, \dots X_n$ are independent and identically distributed, with $E(X_1) = \mu < \infty$, then:

$$\frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{a.s} E(X_1) \tag{6}$$

If $f(\cdot)$ is a continuous mapping and $E\big(f(X_i)\big) < \infty$:

$$\frac{1}{n}\sum_{i=1}^{n} f(X_i) \xrightarrow{a.s} E\big(f(X_i)\big) \tag{7}$$

Compute the integral $\int_a^b g(x)dx$:

$$\int_a^b g(x)dx = \int_a^b \frac{g(x)}{f(x)} f(x)dx = E_f\left(\frac{g(x)}{f(x)}\right) \approx \frac{1}{n}\sum_{i=1}^{n} \frac{g(x_i)}{f(x_i)} \tag{8}$$

Random samples $x_1, x_2, \dots x_n \sim f(x)$ are drawn, and calculations are performed for $\frac{1}{n}\sum_{i=1}^{n}\frac{g(x_i)}{f(x_i)}$. When x lies within the interval $[a,b]$, first sample z on $[0,1]$, then transform $x = b + (b - a)(z - 1)$.

3.4. GARCH Model

Traditional econometrics assumes the second hypothesis that the amplitude of volatility (variance) of time series variables is constant, which contradicts reality. For instance, it's long been recognized that the volatility of stock returns changes over time rather than remaining constant. This renders traditional time series analysis ineffective for practical problems.

The GARCH (p, q) model introduces a relationship for the instantaneous volatility σ_t:

$$\sigma_t^2 = \alpha_0 + \alpha_1 d_{t-1}^2 + \cdots + \alpha_q d_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 \tag{9}$$

4. Empirical Analysis

The closing index of the Shanghai Composite Index is used to simulate the changes in the stock market and stock option pricing. First, extract the closing index of the Shanghai Composite Index from January 1997 to the present, and calculate the logarithmic return rate for each month compared to the previous month:

$$\text{Monthly Return Rate} = \ln\left(\frac{\text{Price This Month}}{\text{Price Last Month}}\right) \tag{10}$$

Generate the trend chart of monthly return rates for the Shanghai Composite Index: (x-axis represents year and month, y-axis represents return rate)

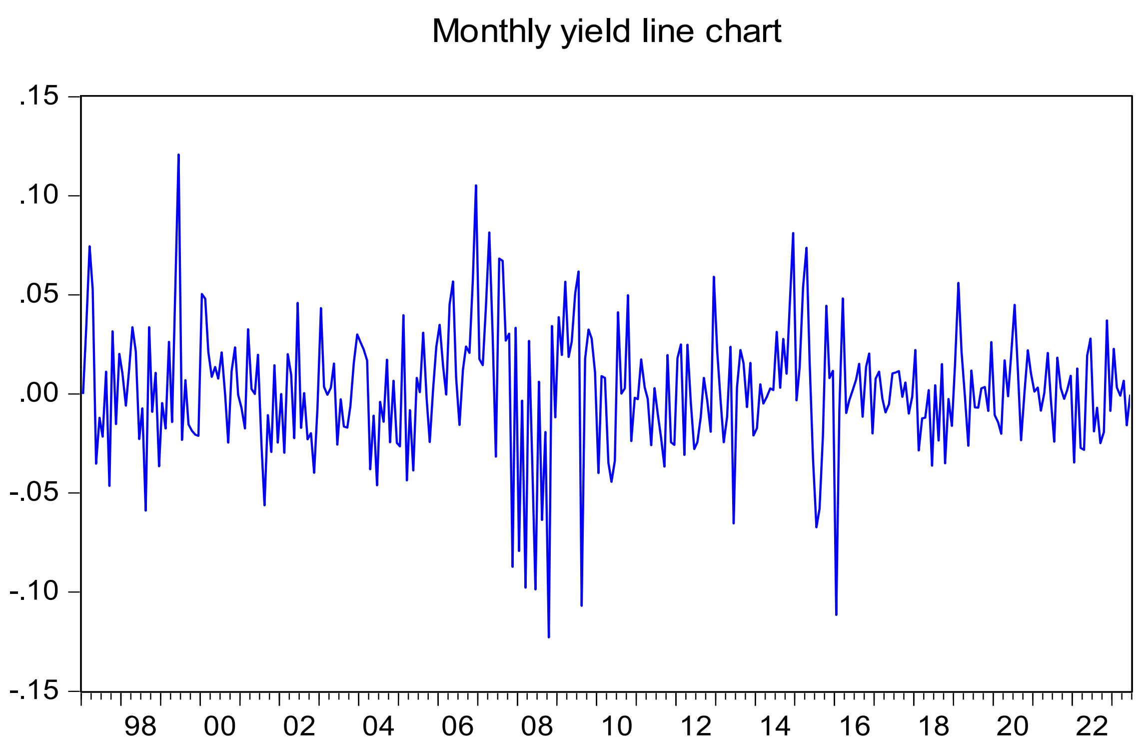

Figure 2: Trend chart of monthly return rates for the Shanghai composite index.

Analyze the changes in the monthly return rates of the Shanghai Composite Index and use the GARCH model to analyze the volatility variance of the index. Estimate the index's volatility, calculate the monthly volatility, generate a sequence of volatility, and plot it as a line chart.

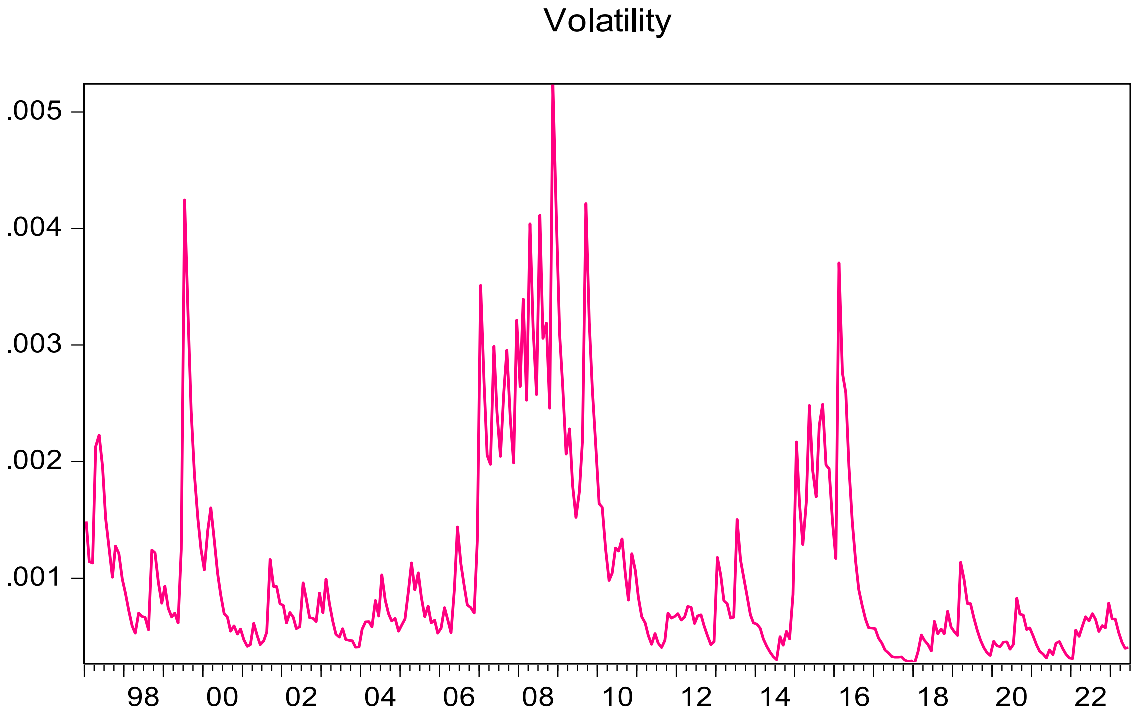

Figure 3: Monthly volatility of the Shanghai composite index.

Table 1: Analysis results of the GARCH model.

Dependent Variable: SERIES01				
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)				
Date: 08/18/23 Time: 18:18				
Sample: 1997M01 2023M06				
Included observations: 318				
Convergence achieved after 21 iterations				
Coefficient covariance computed using outer product of gradients				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
	Variance Equation			
C	6.13E-05	2.72E-05	2.251367	0.0244
RESID(-1)^2	0.224764	0.051195	4.390328	0.0000
GARCH(-1)	0.725687	0.054815	13.23893	0.0000
R-squared	-0.002698	Mean dependent var		0.001638
Adjusted R-squared	0.000455	S.D. dependent var		0.031590
S.E. of regression	0.031583	Akaike info criterion		-4.233169
Sum squared resid	0.317197	Schwarz criterion		-4.197678
Log likelihood	676.0739	Hannan-Quinn criter.		-4.218994
Durbin-Watson stat	1.780501			

The analysis reveals significant differences in volatility across different years, with higher volatility during 2006-2010 and 2014-2016, indicating periods of increased market activity. The overall mean volatility is 0.001638. According to the Black-Scholes pricing formula and the Greek letter vega, options prices corresponding to the market during these periods should also be higher, and option prices are more sensitive to fluctuations in stock prices. Given the heightened uncertainty in today's internal and external environments, the volatility of the stock market might be influenced. Thus, studying historical A-share market volatility is highly valuable for the options market. Volatility and option prices are closely related, as reflected in the Greek letter Vega. Their relationship is depicted in the following chart:

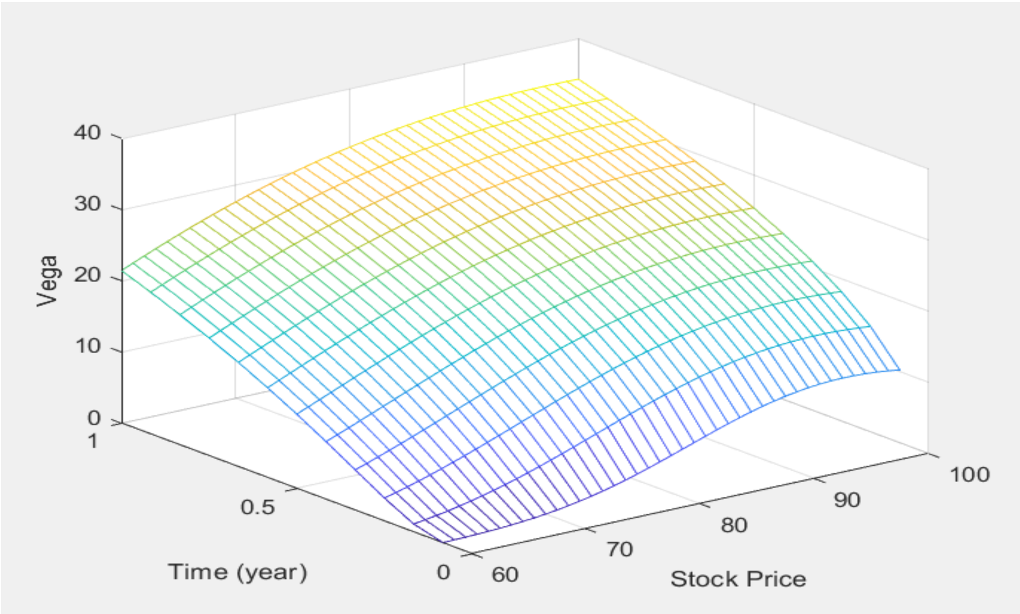

Figure 4: Relationship between vega and stock price and time.

A similar treatment is applied to the SZSE Component Index. Extract the closing prices of the SZSE Component Index from May 1998 to June 2023, calculate the logarithmic return rate for each month, apply the GARCH model to calculate the volatility sequence based on the logarithmic return rate, and plot it as a line chart.

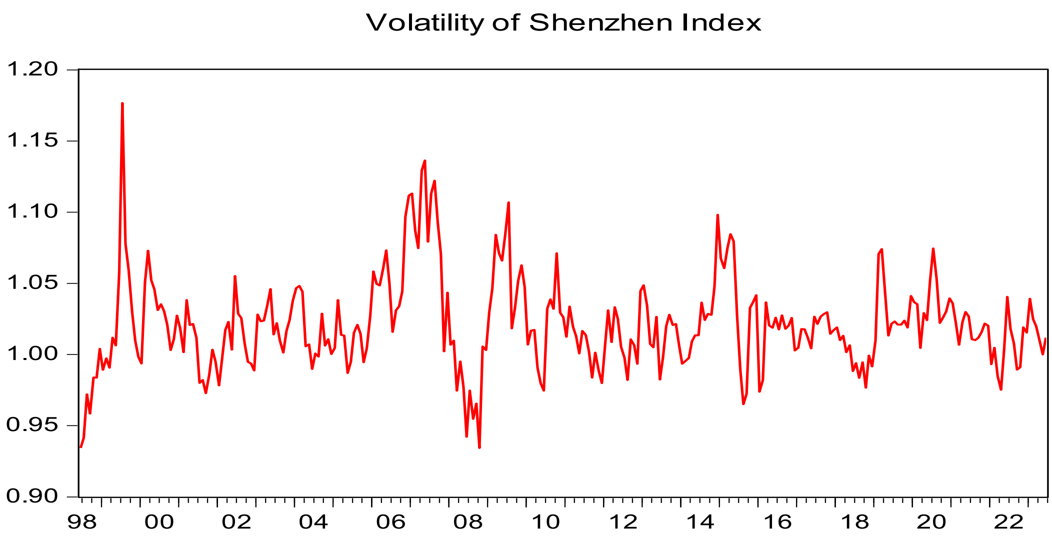

Figure 5: Volatility of the SZSE component index.

Observations indicate that the SZSE Component Index has higher volatility, yet the monthly fluctuations are not as significant as those of the Shanghai Composite Index, suggesting internal

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differences within the securities market. The Shanghai Composite Index generally better represents the overall A-share market performance, while the SZSE Component Index reflects the activity of a subset of stocks. By considering both indices, two peaks in volatility align with the same years, reflecting the similarity in volatility between the two indices.

A similar method is applied to calculate the volatility of the ChiNext Index, and the line chart is shown below.

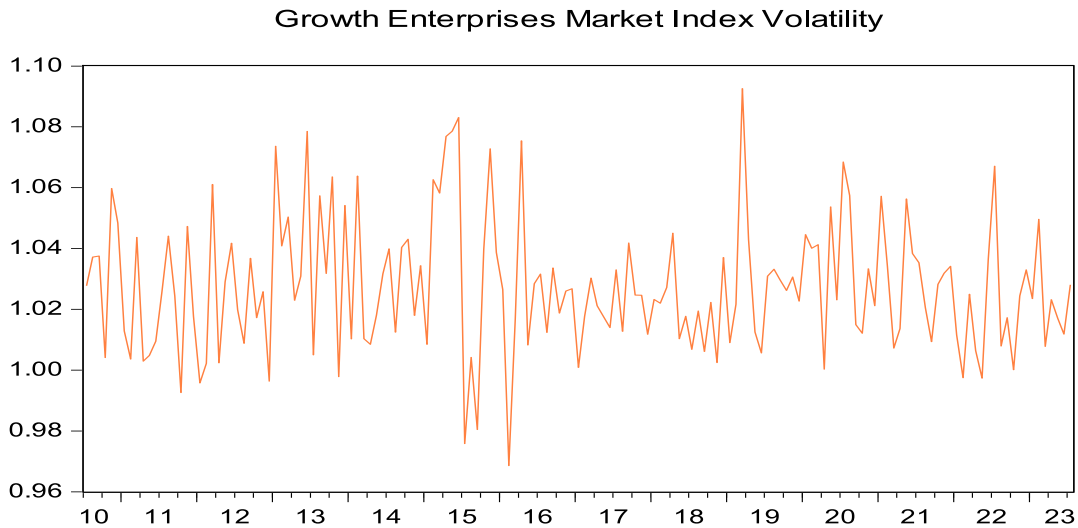

Figure 6: Volatility of the ChiNext index.

The volatility of the ChiNext Index from 2010 to 2023 shows smaller changes, within 10%. This may be related to index calculation methods and national policies. A volatility peak also appeared around 2015, indicating consistency in the market's volatility response across the three major indices.

For the ChiNext ETF Fund (159915), which has underlying options, the closing price is calculated for volatility and return rate using the same method, and the chart is plotted.

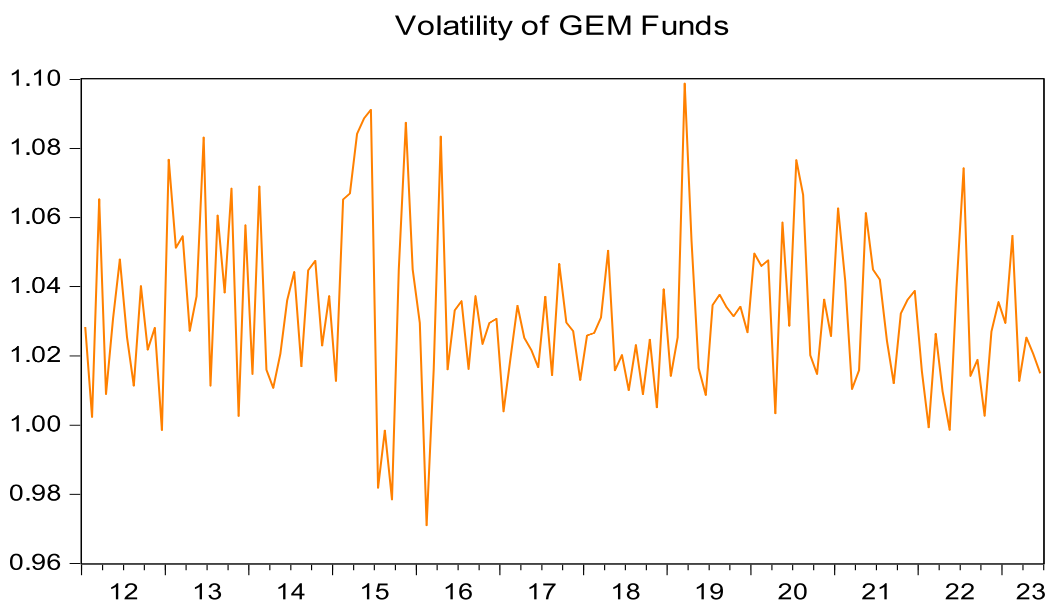

Figure 7: Volatility of the ChiNext ETF fund.

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Due to the close correlation between ETF fund prices and indices, their volatility is almost identical to that of the ChiNext Index.

A similar process is applied to the SSE 50 ETF Index Fund, using annual data for simplicity. The closing price on the last day of each year is used as the dataset.

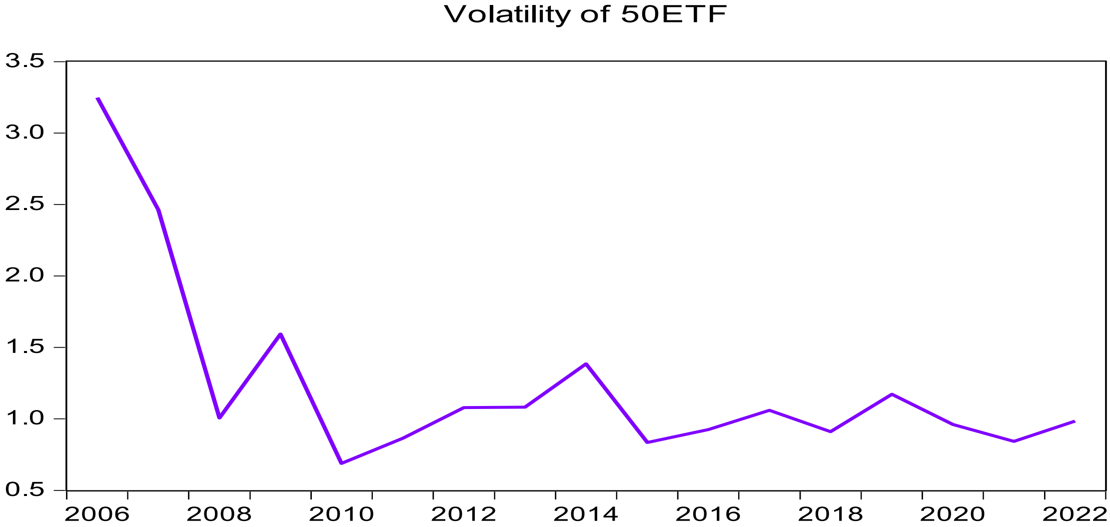

Figure 8: Annual volatility of the SSE 50 ETF index fund.

The SSE 50 ETF Index Fund has higher volatility compared to the Shanghai Composite Index, with larger inter-year changes. However, the cycles of volatility change are relatively stable, and there are no prominent peaks. Peaks occurred around 2009, 2014, and 2019, reflecting differences between index funds and overall market trends. This explains the advantage of index funds as option underlyings.

Using the same method, monthly closing prices of the SSE 300 ETF Index Fund are analyzed for returns and volatility. Monthly closing prices:

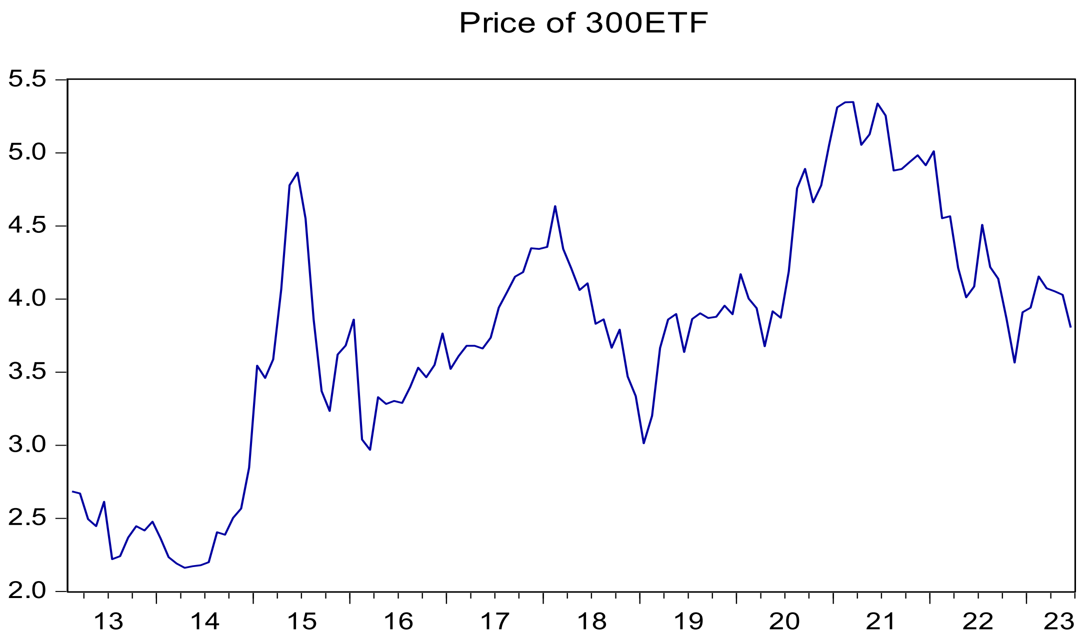

Figure 9: Monthly prices of the SSE 300 ETF index fund.

Volatility:

Table 2: GARCH model analysis results for the SSE 300 ETF index fund.

Variable	Coefficient	Std. Error	z-Statistic	Prob.
	Variance Equation			
C	0.000590	0.000274	2.155743	0.0311
RESID(-1)^2	0.238765	0.091454	2.610769	0.0090
GARCH(-1)	0.633472	0.093186	6.797920	0.0000
R-squared	-0.002160		Mean dependent var	0.002961
Adjusted R-squared	0.005858		S.D. dependent var	0.063962
S.E. of regression	0.063774		Akaike info criterion	-2.714741
Sum squared resid	0.508395		Schwarz criterion	-2.646861
Log likelihood	172.6713		Hannan-Quinn criter.	-2.687165
Durbin-Watson stat	1.706021			

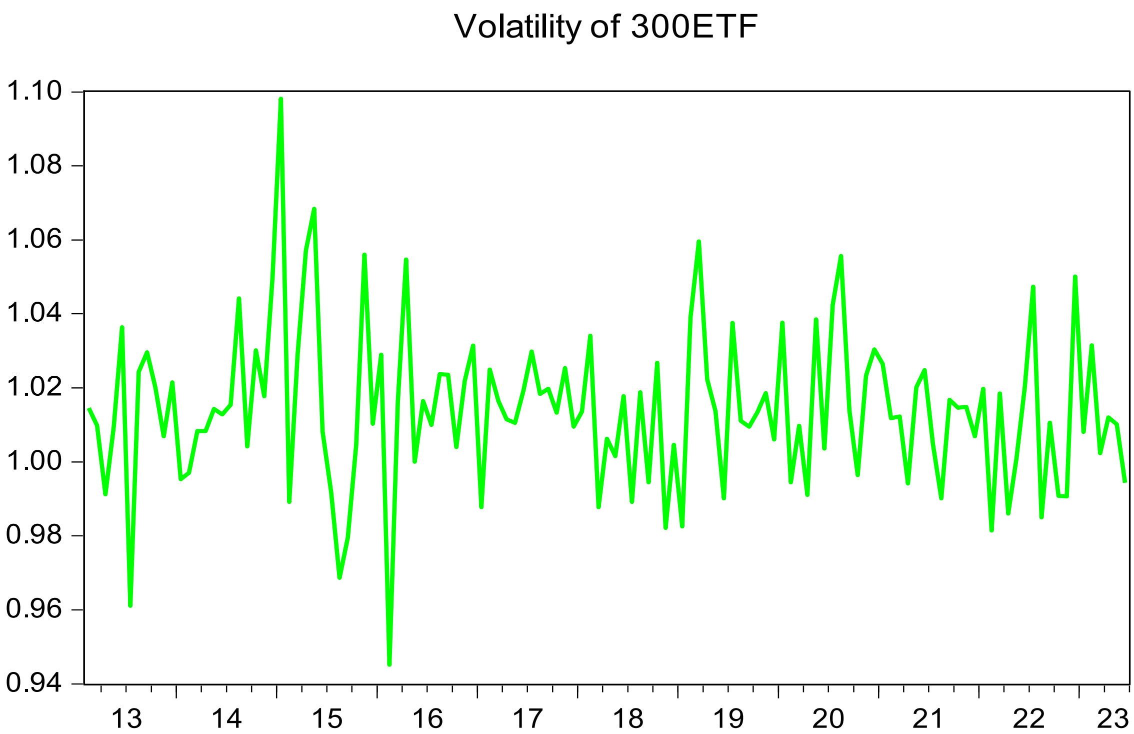

Figure 10: Monthly volatility of the SSE 300 ETF index fund.

The volatility of the SSE 300 ETF remains around 1.00, with relatively minor fluctuations over time. The variation in the past three years does not exceed 5%. The SSE 300 ETF Index Fund has underlying options and exhibits stable volatility, making it suitable for simulating option pricing using the option pricing formula. Given the stable volatility in the past three years, the monthly volatility over this period is calculated and the average value is taken as the overall volatility estimate for this fund:

$$\sigma = \overline{\sigma} = 1.0128 \tag{11}$$

The average price for the first half of the year 2023 is used to simulate the current price of the underlying asset:

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$$S = \bar{S} = 3.9998 \tag{12}$$

Three different strike prices are considered: In-the-money K1 = 3.0, At-the-money K2 = 3.9998, and Out-of-the-money K3 = 5.0. The expiration time is set to T = 3, and the risk-free interest rate is taken as the annual deposit rate r = 0.0175/12 ≈ 0.00146.

Using the Black-Scholes formula, the option prices for the 300 ETF are calculated in Matlab, with the following results:

Table 3: Results of option pricing using the BS formula.

>>BScall(3.9998,3.9998,3,0.0146,1.0128)
Y=2.4815

Using the Monte Carlo algorithm to simulate 10 million iterations for numerical integration, the results for European call options are as follows:

Simulated price, variance, and upper and lower bounds:

Table 4: Option pricing results using the Monte Carlo method.

>>BScall(3.9998,3.9998,3,0.0146,1.0128)		
2.4698	2.4592	2.4559
0.1634	2.4804	2.4837

Table 5: Simulation results for different strike prices using two methods.

Strike Price	3.0	3.9998	5.0
Option Value (BS)	2.6936	2.4815	2.3086
Option Value (Monte Carlo)	2.6992	2.4698	2.3095

Comparison of the two methods reveals that the Monte Carlo method produces results very close to the pricing results of the BS formula through multiple experiments. Sensitivity analysis is conducted by altering parameters. When the expiration time T is changed to 6, the price differences between at-the-money, out-of-the-money, and in-the-money options simulated using the Monte Carlo method narrow. This indicates the influence of time value, implying that when holding options for an extended period, intrinsic value will be less significant than the impact of time value on options, reflecting the meaning of the Greek letter theta.

5. Conclusion and Implications

This study analyzed the volatility of various underlying funds related to the three major stock indices and simulated option pricing. By calculating representative indicators of China's stock market, the volatility's changes over time were validated. The similarities and differences in volatility among different indices and underlying assets were analyzed. The results revealed that, under the condition of calculating monthly price volatility, for certain securities indicators, the volatility did not exhibit significant fluctuations over time, remaining within a range of ±10% around the mean. Examples include the SZSE Component Index, ChiNext Index, and the SSE 300 ETF Fund. In contrast, other securities, such as the SSE Composite Index and the 50 ETF Fund, displayed much greater volatility, reflecting internal variations within the stock market. For the category of securities with significant monthly volatility changes (Δσ), directly substituting the mean volatility into the BS formula for option pricing is inaccurate. For these securities, short-term volatility or instantaneous volatility holds greater reference value. Conversely, for the category with relatively stable volatility, using the BS pricing formula provides a more accurate approximation of option pricing, aligning closer with actual conditions. Regarding the simulation of option pricing for the SSE 300 ETF Fund, altering the time parameter within the pricing formula showed that as the expiration time increases, the differences in option prices for various strike prices diminish. This implies that the fund's high volatility and substantial price changes render the market more active and the options possess a higher time value.

On the other hand, the volatility changes in major indices and funds exhibit similarities, reaching peak points during the same periods. Years with active market trading consistently witnessed higher volatility, demonstrating a certain degree of cyclicality. Returning to the theme of uncertainty, recent years have seen intensified economic policy uncertainty. Nevertheless, the study found that the volatility of major indices has not increased compared to historical years; rather, it has remained relatively stable. This is a shared characteristic among them. This suggests that, currently, the impact of global uncertainty on the stock market is somewhat limited. Influenced by the expansion of market size and policy interventions, the stock market's volatility has remained relatively stable over the past three years. These findings provide insights for understanding the future changes in market volatility, the direction of the options market, and investment choices. The future options market will continue to present both opportunities and risks. Whether in rapidly changing markets or in markets that are becoming more stable, understanding market patterns and adapting to them is essential for optimizing investment returns.

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