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The Difficulties and Optimization of Marriage Matching: An Analysis in the Context of China's Modern Economy

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Abstract: This study delves into the use of matching theory in addressing the stable marriage problem, examining ways to optimize overall utility for both women and men across all stable matchings. The paper introduce the concept of energy, denoted by the ranking where a lower rank signifies higher priority, as a metric for individual satisfaction in a match. The primary objective is to identify a stable match that reduces the average energy expended by both genders. By emphasizing average energy, this study aims to pinpoint a stable match that optimally minimizes this metric. A variety of optimization algorithms and mathematical models are utilized to navigate the solution space and determine the best match. A comprehensive analysis of stable matchings, alongside their associated energies for men and women, unveils the inherent trade-offs and dilemmas stemming from individual preferences. The essence of stability is also explored, underscoring the imperative of balancing individual contentment with the durability of the match. The paper concludes by accentuating the necessity for continued research to bolster overall utility in stable matchings, encompassing the investigation of supplementary constraints and preferences.

Keywords: Marriage matching, deferred acceptance algorithm, average energy

1. Introduction

Marriage is widely regarded in contemporary society as a pivotal milestone in an individual's life. Beyond its emotional underpinnings of love, affection, and responsibility, marriage holds substantial legal significance, warranting societal recognition. The stability it offers not only anchors social order and harmony but also delivers manifold benefits that contribute positively to societal welfare. Moreover, its importance extends to the realm of economic development. Beyond merely supplying society with essential workforce and productivity, marriage also facilitates economic growth through avenues such as the continuation of family businesses.

In recent years, the dynamics of marriage in China have undergone significant transformation. Not only have marriage registration rates declined and divorce rates escalated, but the very structure and perception of marriage have also evolved. Consequently, this study seeks to leverage algorithms and models to foster more stable marital pairings and to enhance and refine the said algorithm further. The stable marriage problem represents a quintessential optimization conundrum, aiming to match individuals from two distinct groups, such as men and women, in a manner that augments their collective contentment. This dilemma surfaces when individuals harbor preferences and need to be matched with counterparts who have their own set of preferences. The ultimate objective is to identify

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a stable match where no pair of individuals would rather choose each other over their designated partners. The Gale-Shapley algorithm, often referred to as the Deferred Acceptance (DA) algorithm, stands as a prominent solution to the stable marriage problem. This method ensures a stable match by iteratively facilitating proposals and subsequent acceptances or rejections until equilibrium is achieved. The Stable Marriage Problem (SMP) holds a paramount position in combinatorial optimization, inspiring an array of research and mathematical progression. It offers profound perspectives on the intersection of fairness and optimality in matching conundrums and perpetually serves as a focal point of inquiry across diverse disciplines. However, the challenge lies in the fact that the most optimal pairing may not inherently be the most stable, hence presenting a dichotomy between individual contentment and overall stability. This essay predominantly hones in on strategies to strike a harmonious balance between these two facets and delves into the real-world applications of SMP in various markets.

2. Literature Review

The stable marriage problem has garnered significant attention across a myriad of disciplines, including but not limited to computer science, mathematics, and economics. Pioneers from these domains have enriched this field through their exploration of diverse algorithms, mathematical frameworks, and pragmatic applications in market design. The DA algorithm, initially conceived as a theoretical solution to the marriage conundrum, has cemented its place as a foundational pillar within this discourse. Through meticulous validation, scholars have underscored both the imperative of a stable solution and its inherent non-uniqueness [1]. Nevertheless, a notable shortcoming of the DA algorithm is its predisposition towards favoring one party over the other, thus sidelining the essence of mutual stability. This discerned gap has catalyzed subsequent scholarly endeavors to refine the existing model.

In 1985, Parisi embarked on a comprehensive study aimed at maximizing overall contentment within stable marital matchings. Employing the replica technique, the study achieved a globally optimal solution. The rank of each individual when matched with a spouse equated to $2\sqrt{N}$, where N denotes the aggregate count of couples in stable matchings [2]. This innovative approach not only addressed the aforementioned unilateral bias but also accentuated the need to prioritize the preferences and contentment of both partners within a pairing. Building upon this, Gusfield and Irving further refined the model, introducing the 'Equitable Stable Marriage Problem'. This model sought to eliminate disparities between the two involved parties by minimizing the absolute variance in total energies of men and women within a given matching M [3]. Nevertheless, Kato subsequently demonstrated that this particular problem is NP-hard, indicating that no existing algorithm can offer a superior solution [4].

While the stable marriage problem has long been a central research concern, Roth's investigations broadened the scope, encompassing market inefficiencies in the U.S. physician landscape. Through the meticulous analyses, Roth pinpointed historical market deficiencies and postulated corrective measures. A standout intervention was the institution of a centralized exchange, reminiscent in its mechanisms to Gale and Shapley's deferred acceptance algorithm [5]. This consolidated platform demonstrated efficacy in rectifying market inefficiencies, optimizing the alignment process not only between doctors and hospitals but also between students and educational institutions, and organ donors with their respective recipients.

Roth's seminal contributions to market design have been transformative, recalibrating existing paradigms and pioneering more effective market mechanisms. By incorporating nuanced alterations to the Gale-Shapley algorithm, Roth's endeavors judiciously acknowledged context-driven needs and ethical imperatives, like eschewing transfer payments. Such enhancements not only augmented market operational efficacy but also fortified principles of justice and equitability within the matching

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framework. This trailblazing approach birthed a distinct economic research subfield known as market design. This discipline is dedicated to sculpting optimal market systems, judiciously weighing various constraints and prerequisites.

Venturing beyond market design, Roth also made notable forays into the realm of the bargaining model. In 1991, underpinned by the empirical investigations, the study introduced a bargaining model. To account for and temper subjects' innate risk aversion, lotteries were strategically employed as stimuli [6]. A subsequent design in 1979, collaborated on with Malouf, refined these experiments by leveraging lotteries to encapsulate shifts in data relayed through one participant's payment to another [7]. Collectively, these experimental pursuits underscored the pivotal role of focal determinants and considerations of equitability in bargaining scenarios.

Ochs and Roth, in 1989, sought to validate the prognostications of the noncooperative bargaining model through a series of structured experiments [8]. These investigations yielded enlightening observations regarding the evolution of subjects' behaviors in consecutive bargaining contexts. Augmenting this understanding, Ro and the colleagues embarked on a cross-cultural inquiry in 1991, scrutinizing bargaining practices across four distinct nations. This study illuminated the profound impact of cultural nuances on the eventual outcomes of bargaining sessions [9].

Fast forwarding to 1995, Roth, in collaboration with Erev, formulated a reinforcement learning model, elucidating participants' propensity to replicate decisions that have historically borne favorable results. This model's predictions neatly aligned with observed behaviors across a spectrum of empirical games [10]. Later, in 1998, Slonim and Roth harnessed this model to interpret a non-cooperative bargaining game, showcasing the prescient capacity of integrated learning frameworks [11]. These seminal works underscore a salient point: recognizing the cognitive confines intrinsic to real-world scenarios can significantly bolster both the explanatory and anticipatory prowess of game theory.

Drawing inspiration from the remarkable strides made by these scholars, the discourse will progressively delve deeper into the stable marriage problem and its interplay with tangible market dynamics.

3. Methodology and Discussion

Contemporary marriage challenges underscore that even stable matches harbor inherent instabilities. Additionally, the rising phenomenon of singlehood in China presents significant demographic disparities. Drawing from the sixth population census of 2010, the nation reported approximately 22.79 million unmarried individuals. Intriguingly, unmarried men constituted 57% of this figure, while unmarried women represented the remaining 43% [12]. Theoretically, these instabilities can be attributed to the variances in how men and women prioritize their potential partners. Greater congruence in these rankings typically heralds enhanced relationship stability, given both partners approximate or match each other's ideal preferences. However, pragmatically, a myriad of intricate moral, ethical, and societal factors muddy these waters. Consequently, partner matching becomes highly unpredictable, influenced by factors like familial income and educational backgrounds, among others. Therefore, the investigative focus remains anchored to the Gale-Shapley (GS) algorithm, aiming to derive insights that straddle both theoretical frameworks and tangible realities. Furthermore, this study delves into the interplay between the foundational stable marriage problem, intrinsic to matching economics, and its ramifications and contributions to tangible challenges, notably in the domains of market design and planning.

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3.1. Correlation between Individual Utilities for Men and Women

To delve into this correlation, the paper employs the term "energy" to illustrate the inherent trade-offs in the stable matching process. This "energy" can be conceptualized as the least preferable ranking a man or woman would consent to, and its calculation is facilitated by the GS algorithm. For clarity, let's consider a basic example where N=3 men are paired with N=3 women.

The preferences of men are listed as:

(1) Man 1's order: 1, 2, 3

(2) Man 2's order: 1, 3, 2

(3) Man 3's order: 2, 3, 1

Conversely, the preferences of women are:

(1) Woman 1's order: 3, 1, 2

(2) Woman 2's order: 1, 2, 3

(3) Woman 3's order: 3, 2, 1

From these preference lists, the "energy" or rank trade-off for each individual is determined. For instance, if man 1 is paired with woman 2, the energy value is 2, reflecting its second-place ranking on the list. Similarly, woman 2 has an energy of 1 in this matching scenario.

Using the Gale-Shapley algorithm, the man-optimal stable matching for this instance is ((1, 1), (2, 3), (3, 2)). Conversely, the woman-optimal stable matching is ((1, 1), (2, 2), (3, 3)). When assessing the energies for men and women in these matchings, the study reveals that the average energy of men in the man-optimal stable matching is computed as (1+2+1)/3 = 4/3, while the average energy of women stands at 7/3. In the woman-optimal stable matching, the average energy for men is 6/3, and for women, it's 5/3.

The study introduces definitions F and H for women and men respectively. Here, f (w, m) signifies the position of a man m in the preference list of a woman w. Similarly, h (m, w) can be interpreted in the same manner. Based on the energy definition, an equation can be formulated to determine the average energy per individual, both for women and men, within a specific matching as:

$$\varepsilon_{F}=\frac{1}{N}\sum_{i=1}^{N} f(w_i, m_i) \tag{1}$$

$$\varepsilon_{H}=\frac{1}{N}\sum_{i=1}^{N} h(m_i, w_i) \tag{2}$$

Where N represents the total number of couples. From prior understanding, it's evident that a reciprocal relationship exists between the energies of men and women: when one increases, the other decreases, and vice versa. To elucidate this, let's consider a theoretical scenario wherein a stable matching exists, and a specific woman, denoted as woman w, has an energy of ϵw. In such a setting, the optimal strategy for women would be to decline all proposals from men who rank above ϵw. For men, the ideal strategy remains consistent with the Gale-Shapley algorithm.

Examining the intricacies of this adapted algorithm, let's consider a man, m, and the proposal to a woman, w. If the woman accepts the offer, they establish a pair (m, w). Conversely, if the woman declines the proposal, the man advances to the next woman on the preference list. This procedure persists until the man encounters a woman who ranks below ϵw, prompting to cease the proposals.

Given the previously mentioned strategy, the likelihood of a man's proposal being accepted stands at ε_w/N. Consequently, an equation can be formulated that represents the probability of this man securing the q^{th} choice.

$$P_H(q)=\prod_{w=1}^{q-1}\left(1-\frac{\varepsilon_w}{N}\right)\frac{\varepsilon_q}{N} \tag{3}$$

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The probability can be represented by a geometric distribution, with H modelled as H ~ Geo ($\frac{\varepsilon_w}{N}$). Given the prior details, where ε_w is an independent variable, this study denotes ε_F as the average of ε_w. With this understanding, the previous formula can be reformulated as:

$$P_H(q)= (1 - \frac{\varepsilon_F}{N})^{q-1} \frac{\varepsilon_F}{N} \quad (4)$$

From equation (4), the average energy for a man can be computed, $\varepsilon_H=\sum qP_H(q)$. Given that this is also the expected value of event H within a geometric distribution, the resulting equation is:

$$\varepsilon_H=N/\varepsilon_F \quad (5)$$

This leads the paper to deduce that, in a random instance of the stable marriage problem, the energies for men and women are reciprocally related. Figure 1 denoted by a solid line, further elucidates equation (5), where it displays the distribution of individual energies of men (represented by open circles) and women (indicated by full circles) across all stable matchings in an instance where N=1000. This is presented on a logarithmic scale [13].

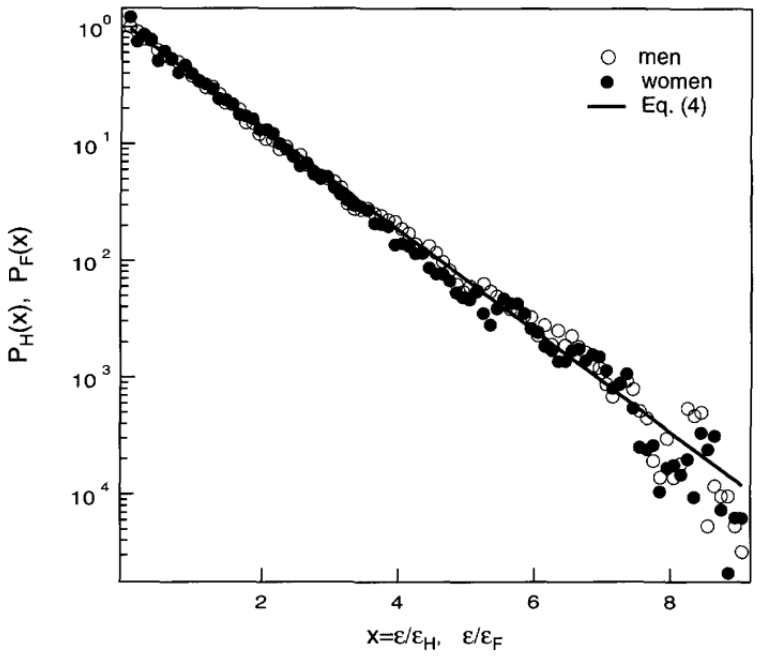

Figure 1: Distribution of individual energies of men (represented by open circles) and women (indicated by full circles) across all stable matchings in an instance where N=1000 [13].

3.2. Application of a Matchmaker and the Role of Information

In the theoretical model, the paper deduces that when a man and a woman are paired without constraints, their individual utilities are inversely proportional. This concept is mirrored in the real world, particularly on certain matrimonial websites and applications. Here, intermediaries or matchmakers facilitate pairings, aiming for the most ethically informed matches possible.

One method to address the Stable Marriage Problem (SMP) involves deploying a matchmaker. This entity possesses comprehensive knowledge about all participants and aids the matching progression. The overarching objective of the matchmaker is to augment societal benefits while also pursuing its personal gains.

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Within the SMP framework, the matchmaker recommends pairings to participants grounded in their preferences. This strategic approach ensures the efficacy of matches and cultivates elevated satisfaction for everyone involved. The matchmaker's revenue stems from commissions paid by the participants, and these earnings correlate directly with the collective societal benefit.

It's crucial to recognize that the motivations behind a matchmaker's actions aren't rooted purely in altruism; self-interest plays a significant role as well. Nonetheless, in a meticulously crafted matching system where the matchmaker operates transparently and impartially, both immediate gains and future benefits can harmoniously serve both individuals and the broader society.

The realm of matchmakers extends beyond just the SMP. In today's dynamic markets, numerous informational entities, colloquially termed "matchmakers," guide consumers in pinpointing the most suitable products or services. Leveraging state-of-the-art techniques like big data analytics, these matchmakers curate product suggestions tailored to consumers, streamlining their decision-making processes. Renowned platforms exemplifying this matchmaker model encompass TripAdvisor, Amazon, eBay, and Alibaba.

Matchmakers hold pivotal roles in modern markets. Given the vast array and diversity of products, they offer a beacon to individual consumers who might otherwise find it challenging to sift through options and make informed decisions. These intermediaries bridge the informational chasm between consumers and companies, aiding consumers in assessing their choices and honing their decision-making prowess. Concurrently, matchmakers afford businesses the prospect of precision-targeted advertising.

However, it's essential to underscore that matchmakers might not consistently maintain impartiality. They could harbor specific biases or inclinations that color their suggestions. In certain instances, these entities might favor the interests of one side due to economic inducements or heightened negotiation leverage of certain stakeholders. In an ideal setting, matchmakers would work assiduously to optimize benefits for both consumers and firms alike. Thus, it's imperative to devise business frameworks that equitably serve both parties.

3.3. Discussion

In conclusion, the stable marriage problem introduces a multifaceted optimization predicament. It demands a delicate balance between individual contentment and the overarching stability of the matches. The metric of "energy," emblematic of individual preferences, serves as a tangible gauge for the satisfaction levels of both men and women within a pairing. Through a keen evaluation of the mean energy accumulated by both genders, researchers aspire to identify a stable match that optimizes collective benefit.

A plethora of optimization algorithms and mathematical frameworks are at the disposal to navigate the solution landscape and pinpoint the ideal match. Tools such as genetic algorithms, simulated annealing, and linear programming are potent in their capacity to iteratively scout for superior matches. Central to these methodologies is their inherent design to factor in individual preferences and stipulations, ensuring the discovery of a stable match that curtails the mean energy expenditure.

The exploration into stable matchings and the associated energies for both genders unveils inherent trade-offs and dilemmas rooted in personal preferences. This study elucidates that while stable matchings ensure that no participant finds incentive in deviating from their chosen partner, these matchings often don't represent the universally optimal solution in terms of individual contentment.

Drawing parallels, the stability emphasized in the stable marriage problem resonates with the Nash equilibrium in game theory. Here, participants fine-tune their personal contentment while keeping others' actions in the backdrop. Yet, this investigation underscores a pressing need to transcend mere stability. It promotes the idea of holistically assessing the mean energy amassed by both genders as a pathway to a more universally favorable outcome.

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In essence, the stable marriage problem poses an intricate optimization puzzle, harmonizing individual inclinations with the overarching imperative for stability. The focus on the average energy of both genders aims to derive a stable match that epitomizes collective utility. The research not only brings to the fore the conflicts and compromises inherent to personal preferences but also accentuates the urgency to delve deeper into optimization algorithms and mathematical paradigms, aspiring to heighten overall contentment within stable matchings.

4. Limitations and Outlook

The enhancements made to the delayed acceptance algorithm bolster its capability to consistently produce stable matches, irrespective of participants' preferences. Additionally, its strategy-proof nature means participants can't game the system to their advantage. However, while the delayed acceptance algorithm is impeccable within specific parameters and minimal external interferences, its application across diverse social activities is restricted. While mathematics offers profound insights, it can't replace the nuances of economics or the intricate realities of human choices. It's impractical to map all matrimonial decisions to a mere mathematical representation of 'energy'. Factors like age, cultural background, familial circumstances, and shared interests play a pivotal role. Although economics encourages rational decision-making, it's evident that human relationships extend beyond simple calculations. Yet, if there's a key takeaway from this theoretical framework, it's the empowerment of taking the lead, as initiators often secure matches with the lowest energy costs.

With the development of the Internet and mobile connectivity, interpersonal interactions have become more accessible and unrestrained, facilitating the search for like-minded partners. Future marriages are anticipated to prioritize emotional bonds and mutual support over the conventional motives of economic gain and procreation. As society gains a deeper understanding of marital relationships, there may be a stronger emphasis on the quality and longevity of unions. This will likely result in increased focus on communication, tolerance, and mutual support as foundational elements for maintaining enduring, healthy relationships. For instance, accounting for factors like age, education, or socioeconomic backgrounds might offer a more genuine reflection of the real-world preferences and limitations individuals encounter. Delving into the influence of additional preferences, such as shared interests or values, could further enhance the fulfillment derived from stable matchings.

5. Conclusion

This study expands upon the foundational deferred acceptance algorithm by integrating the concept of energy, aiming to enhance the overall utility of stabilized pairings. From this exploration, three principal conclusions emerge. Firstly, minimizing average energy is instrumental in ascertaining the optimal global pairing. Secondly, the energies associated with different genders are reciprocally related. Lastly, the ranking of the most compatible partner remains consistent at twice the square root of N. Historically, the stable marriage problem has been rigorously examined across multiple disciplines. Scholars have delved into diverse algorithms, mathematical paradigms, and optimization strategies to identify stable matches that optimize collective contentment. Notably, the principles of stabilization, blocking pairs, and rotation are pivotal in deciphering the framework and attributes of stable matchings. This study also sheds light on the scaling nature of stable matchings and the equilibrium between individual satisfaction and stability.

Furthermore, this research suggests a direction for future inquiries to delve into the scalability and adaptability of the problem, enhancing its applicability to real-world contexts. Indeed, the quest for stable matches within marital contexts remains a vibrant and consequential field of study. Scholars

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are consistently crafting innovative algorithms and systems, progressively introducing nuanced constraints and preferences to identify the ultimate match. Enhancements to the deferred acceptance algorithm have augmented its adaptability, efficiency, and efficacy, making it indispensable for myriad matching challenges. This algorithm persists as a critical instrument across various sectors confronting matching dilemmas, yielding pragmatic solutions that culminate in stable and gratifying results.

Authors Contribution

All the authors contributed equally and their names were listed in alphabetical order.

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