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The Valuation of Internet Companies Based on Adjusted Black-Scholes Model

Yanbin Yang[1,a,*]

[1]University of Rochester, New York State14604, USA
a. sryang569@gmail.com
**corresponding author*

Abstract: With the continuous breakthrough of Internet technology, we can clearly see the huge growth space and potential profitability of related companies. However, due to their unique instant change and uncertainty, high returns and high risks exist simultaneously and finding new approaches different from traditional ones to evaluate internet companies is necessary and significant. Based on adjusted Black-Scholes Model, this study discusses three approaches: Metcalfe approach values internet companies that relies on user numbers. DCF approach and Fuzzy real option approach estimate internet companies with strong cash flow and long-period potential growth, focusing on probability and fuzzy theory separately.

Keywords: Black-Scholes Model, internet company, fuzzy, Metcalfe, valuation

1. Introduction

As the information technology revolution progresses, Internet enterprises occupy an increasingly important position and the internet has become part of our life. However, the public has recently been once again amazed by the development of the Internet since Chat GPT was launched on 30th November 2022. The official definition of ChatGPT is a model which interacts conversationally; it can answer followup questions, admit its mistakes, challenge incorrect premises, and reject inappropriate requests [1]. Stanford news gives a more accurate and accessible description: it has drafted cover letters, composed lines of poetry, pretended to be William Shakespeare, crafted messages for dating app users to woo matches, and even written news articles, all with varying results [2]. Actually, just like New York Times reports that "it has set off a feeding frenzy of investors trying to get in on the next wave of the A.I. boom."[3], 13 million individual active users visited ChatGPT per day as of January 2023 and $200 million in revenue is expected by the end of the year 2023 [4]. From these data, it is clear to see those typical internet companies represented by ChatGPT are different than other normal companies-internet companies; internet companies usually come with tremendous growth have more volatility, having more potential risks and profits. Hence, given the high possible return and popularity of internet companies, it is necessary and meaningful to estimate the value of internet companies in terms of their special characteristics.

However, up to now, there are no universally accepted models to value internet companies accurately in practice because of their special features. According to Forbes[5], three current primary valuation methodologies for internet stock include a discounted cash flow, or DCF analysis consisting of estimating a firm's future cash flows and discounting them back to the present for a current value estimate. The comparable approach consists of using comparable companies and calculating

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comparable multiples. Nevertheless, the biggest problem here with these three classical methods is their theoretical assumptions or significant limitations.: they either usually assume a perpetual growth rate and anything will hold in perpetuity or don't consider the company's EPS growth prospects. For example, even if a company has a very high P/E ratio by a comparable method, investors will still feel comfortable buying it because they believe P/E ratio will be back to a low level. Facing high uncertainty for internet companies in reality, current methodologies have obvious shortages. Therefore, the objective of this paper is to use the approach based on the Black-Scholes Model that is particularly suitable for the valuation of uncertainty to try to better value internet companies. It shows and reviews three different methods based on different adjusted Black-Scholes Model models and their practice use. It's to help all investors or organizations to better evaluate internet companies and pay up for their outlooks and growth in a reasonable range and avoid the dot-com bubble the next time.

2. The characters and essence of internet companies

According to Britannica [6], the definition of an internet company is a company that provides internet connection and service to individuals and organizations. In fact, today's internet companies have been an important part of the global financial market and provide services for huge users. Their revenue model is different from others: in most cases, they generate profits without charging users for access to content through advertising revenue. For example, Google's over 80% of revenue is from the advertisement in 2020 [7]. Google charges advertisement buyers and let buyers put their advertisements on Google search, YouTube, and other websites to reach all potential consumers. Meanwhile, internet companies usually have other income resources to diversify into other areas. It includes cloud storage, premium account, subscription, and so on. Overall, one successful company can easily earn tremendous profits through its users and the convenience of the internet which breaks the limitation of time and region such as Google, Meta (Facebook), Instagram. Among them, Meta (Facebook) was created by several Harvard students in 2004, and it filed to become a public company in 2012 and eventually has nearly three billion users as of 2021 [8]. Such incredible growth let many investors crazy about internet stocks, but high return always comes with high risk. Many internet companies failed in the dot-com bubble or later. Certainly, it is hard to say what's the essence of surviving successful internet companies or what supports them to be the winner in the fierce competition and high risk, there are at least some common features of good internet companies.

The following features are usually owned by successful or promising internet companies [9]:

- A business must have the potential to grow revenue and gross margin at an attractive rate.
- The business is internationally reachable.
- A product that is sufficiently "sticky" that customers are likely to use it for a long period instead of using others.
- Allow the company to acquire a significant number of customers at a reasonable cost.
- Have a necessary or attractive need.

3. Black-Scholes Model

3.1. The definition of Black-Scholes Model

The Black-Scholes model, also known as the Black-Scholes-Merton (BSM) model, is the cornerstone of modern financial theory. It was created by economists Fischer Black and Myron Scholes and was named after their names. It was the first widely used mathematical method to calculate the theoretical value of an option contract, using six variables: volatility, type, underlying stock price, strike price, time, and risk-free rate. It is based on the principle of hedging and focuses on eliminating risks

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associated with the volatility of underlying assets and stock options [10]. There are several assumptions about Black-Scholes Model. One of the most important assumptions is that the model follows Random Walk Theorem; this is to say, markets are random and market movement can't be predicted. This theory has gradually been proved and accepted by most investors and organizations in practice. Many recent performance studies reiterating the failure of most money managers to consistently outperform the overall market has indeed led to the creation of an ever-increasing number of passive index funds [11]. Meanwhile, the model indeed has other few unrealistic assumptions like other all financial models. However, this problem has been solved to a great extent by using partial derivatives— Option Greeks [12]. Generally, Black-Scholes Model can efficiently hedge and calibrate various assets in the right way and eliminate risk, having great potential use for high uncertainty valuation such as internet companies.

Here are formula and inputs in Black-Scholes Model:

For call options:

$$C(t, S(t)) = S(t)N(d1) - Ke^{-r(T-t)}N(d2)$$

$$N(x) := P[Z \leq x] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}\, dy$$

$$d1 = \frac{1}{\sigma\sqrt{T-t}}[\log\left(\frac{S(t)}{K}\right) + (r + \frac{\sigma^2}{2})(T-t)]$$

$$d2 = \frac{1}{\sigma\sqrt{T-t}}\left[\log\left(\frac{S(t)}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right] = d1 - \sigma\sqrt{T-t}$$

For put options:

$$P(S(t), t) = Ke^{-r(T-t)} - S(t) + C\big(t, S(t)\big) = N(-d2)Ke^{-r(T-t)} - N(-d1)S(t)$$

- C: the price of call option
- P: the price of put option
- S(t): the price of the underlying asset at time t
- t: a time in years
- T-t: the time to maturity
- K: the strike price of the option
- σ: the standard deviation of the stock' return, a measure of volatility
- r: risk-free interest rate

3.2. Case about using Black-Scholes Model for hedge and calibration

We use one of the most famous internet companies-Microsoft as our target and focus on its call option to show how Black-Scholes Model can be used to hedge and calibrate. Before we do a delta hedge, we need to estimate the stock's implied volatility by observing its historical prices, or, even simpler, by calculating other option prices for the same stock at a different time to maturity or different exercise/strike prices under the set of the Black-Scholes model.

Take all assets' closing prices on March 3, EST time as their price, we first choose three call options with different strike prices and the same time to maturity. Sigma (implied volatility) is unknown here and assuming sigma is 0.2 here; it can be adjusted to the optimal value by looking for minimal SSE error under the model. Given time to maturity is 0.503, sigma is 0.2, stock price is 255.29, strike price is 255,260 and 250 separately, we can get Table 1, and SEE error here is 10.99585. To minimize the SEE error, we can adjust the value of sigma and eventually let SEE error minimize to 0.026013 when sigma is about 0.329656. The result based on adjusted sigma is Table 2.

Table 1: Calculation of call options by BS Model before sigma adjustment

Number	Time to matureity	d1	d2	N(d1)	N(d2)	K	S	Call Price
1	0.503	0.07894	-0.06291	0.53146	0.47492	255	255.29	14.5715
2	0.503	-0.05796	-0.19881	0.47689	0.42082	260	255.29	12.33301
3	0.503	0.21854	0.07669	0.5865	0.53057	250	255.29	17.08478

Table 2: Calculation of call options by BS model after sigma adjustment

Number	Time to matureity	d1	d2	N(d1)	N(d2)	K	S	Call Price
1	0.503	0.12176	-0.11204	0.54846	0.4554	255	255.29	23.8893
2	0.503	0.03871	-0.19509	0.51544	0.42266	260	255.29	21.6946
3	0.503	0.20646	-0.02734	0.58178	0.48909	250	255.29	26.5017

After having sigma (implied volatility), we can use it to do delta hedge and calibration. We take one of the other Microsoft options with a different time to maturity (0.624) and hedge it for seven continuous trading days using the calibrated parameters. By creating a portfolio based on this option and the underlying asset, we hold the option and sell Δ(t-1)-Δ(t) shares stock at t day as Table 3 shows. To calculate wealth, also called portfolio value, the formula is X(t) = $X(t-1)$+$\Delta(t-1)[S(X)-S(X-1)]$. The special situation is X(0) = C(0). Eventually, we can get loss in option is 0.29, portfolio value is 26.292, loss with hedging is -26.0019 (which means the total return is positive) and loss without hedging is -24.2374. Since loss with hedging is smaller than loss without hedging, we can indicate that Black-Scholes Model works for internet stock and brings higher profits for investors. Nevertheless, in the face of increasing diversification of revenue models and complicated conditions in the development of internet companies, we need to adjust Black-Scholes Model to value internet companies under ambiguity, uncertainty, and variety from a higher dimension.

Table 3: Portfolio' hedge and calibration by BS Model

Time	Stock Price	d1	d2	N(d1) (Δ)	N(d2)	Call price	Wealth
0	251.51	0.077284	-0.18312	0.530801	0.42735	24.52743	24.52743
1	254.77	0.126739	-0.13367	0.550426	0.446832	26.28993	26.25784
2	249.22	0.042159	-0.21825	0.516814	0.413618	23.32788	23.20298
3	250.16	0.056616	-0.20379	0.522574	0.419258	23.8164	23.68878
4	249.42	0.04524	-0.21517	0.518042	0.414818	23.43137	23.30208
5	246.27	-0.00357	-0.26398	0.498577	0.3959	21.83012	21.67024
6	251.11	0.071172	-0.18924	0.528369	0.424954	24.3156	24.08336
7	255.29	0.134569	-0.12584	0.553523	0.44993	26.57695	26.29194

4. Methodology for assessing the value of internet companies by using adjusted Black-Scholes Model

4.1. Metcalfe approach based on Black-Scholes Model

With the rise of social media companies in the past ten years, the public has gradually been used to interacting with friends or families online because of its easy connection, instant access, and opportunities to reach more people. Up to January 2023, there are 4.76 billion social media users in the whole world, making up 59.4% of the total global population [13] In addition to the huge number of users, time spent on social media is also stunning: people usually spend 15% of their waking time on social programs if we assume the sleep time is between 7 and 8 hours. Hence, one very interesting

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corresponding concept called the network effect was created and later has been promoted by Robert Metcalfe, the father of Ethernet. He conceived Metcalfe's law to better illustrate the network effect, explaining how social media users correlate with the internet itself. This law states that the value and cost of the network are directly linked to the number of connected users of system, and both are proportional to the square of the number of users. An example is that one single Facebook account is meaningless because it can't interact with anyone else. However, as more and more people join Facebook in the internet, the value of each Facebook account increases because they can send and receive messages to others. In another word, the value of social media companies depends on their number of users. Therefore, by plugging Metcalfe's law into Black-Scholes Model, we can effectively quantify the importance of user numbers to the value of Internet companies and provide a new perspective here.

Following Metcalfe's law, we define network size, value, and cost as revenue, operating cost, and MAU (monthly active user) separately. Then according to Metcalfe's law, we can use Python or R studio to do data analysis to find two equations about how revenue and cost relate to MAU. Theoretically, we get equations as below and the variables used in the equations have illuminated in Table 4:

$$V = K1n^2,\ C = K2n^2,\ \text{Netoid} = p/(1 + e^{-v(t-h)}$$

Table 4: Symbols used in Metcalfe's law

Symbol	Definition	Practical meaning
V	Value of network	Revenue
C	Cost of network	Operating cost
n	number of nodes	MAU
Netoid	Growth trend of n	MAU

Definitely, we need some real data to prove this law is available in reality. After the creation of the law, there is no data-based evidence to directly support it for some time. It is reasonable because it was the early stage of internet development. With the establishment and stabilization of social media giants such as Facebook and Twitter, enough data has been collected and Metcalfe later used a generalization of the sigmoid function (netoid) to model Facebook and proved Metcalfe's law can be closely fitted to Facebook's associated revenue by using Facebook's real data from 2004 to 2015 [14]. Later, from the most authoritative academic institution in China- Chinese Academy of Sciences, its researchers applied Metcalfe's law to Tencent and Facebook. Tencent is a social media giant in China and owns QQ and Wechat, having over 1 billion users. They also successfully proved the growth trends of Tencent and Facebook monthly active users fit the netoid function well [15].

After validating the availability of Metcalfe's law, we can move to how to use the law in Black-Scholes Model. To estimate the market value of internet companies, adjusting variables in Black-Scholes Model is necessary. The below part shows variables' new definition in the approach:

- σ: annual volatility
- S: cumulative income discount for t years
- K: cumulative cost discount for t years
- t: life span of the internet company

Given two equations from Metcalfe's law, we can predict future MAU, cost and revenue. Then we can calculate the expected revenue and cost in the following t years and discount them to the present to get S and K. Finally, we plug these variables to get d1, d2, N(d1), N(d2), and C. C here means the estimation of market capitalization. According to related research, estimation of Weibo (the second-largest social media platform) by using Metcalfe's approach is relatively close to the real market value and the deviation rate is about 15%, much less than using the unadjusted Black-Scholes Model [16].

Overall, Metcalfe approach takes into account the influence and value of the number of users and is effective in evaluating internet companies that focus on social media products such as TikTok, Wechat, Twitter and so on. Meanwhile, in consideration of current government policies and situation, we need to notice that sometimes social platforms can't be accessed freely and instantly by the regulation and limitation of local laws. For example, some popular western social software such as Facebook is prohibited in China, but Facebook still has huge transportable global space since it is available in most other countries in the world. Simultaneously, Wechat owned by Tencent has users mostly from China and is not very popular in Europe and America. However, relying on the huge China population, Wechat also gains great transportable space like Facebook. Hence, despite government limitations or regulations, social media companies still follow the law of Metcalfe's law if they have enough available transportable space. In sum, blind viewing of user numbers tends to lead this approach to be misused and we need to see whether the applied social media platform has enough space for flow.

4.2. DCF approach based on Black-Scholes Model

Before the development of internet, in the financial market, if investors wanted to invest in the stock market, they usually needed to look for a brokerage to do it and finished transaction by calling. This traditional model usually charged high commissions and had a requirement for investors' principal. Nevertheless, beginning in the 1990s, more small players joined the market and they were online broker companies driven by the rapid development of industry. Even though traditional brokerage also entered online markets later, these small online broker companies focus on potential small investors and quickly have occupied a large share of brokerage business. To be specific, via the convenience of the internet, investors can trade anytime and anywhere and lower commission fees and account minimums than traditional broker attract a wider range of people. According to CNBC [17], just in January 2021, roughly six million Americans downloaded a retail brokerage trading app, joining well over 10 million Americans who opened a new online brokerage account in 2020. Actually, the rising power of new online investors has enabled online brokerage from the single online transaction model to diversify, turning to a more diversified online bank. For instance, Charles Schwab has gradually set assessment management service and it has been one of the most important revenue sources. For evaluating internet companies with stable cash flow and long-time development strategies and plan such as online brokerage, combination of DCF and Black-Scholes Model is a good choice.

DCF (discounted cash flow) method is one of the most common methods in the evaluation of companies, especially banks. It is a method of estimating the current value of a company based on projected future cash flows adjusted for the time value of money. The core advantage of the DCF method is that it's extremely detailed, includes all major assumptions about the business, and determines the intrinsic value of the company based on future cash flow faithfully [18]. However, the DCF approach ignores market sentiments and doesn't work under uncertainty. For instance, at the beginning of March this year, Silicon Valley Bank, the 16th largest bank in the country with $210 billion in assets, announced its collapse due to inadequate liquidity and insolvency. In fact, it is a healthy bank with enough assets, but its heavy exposure to venture capital and tech sector let it fall down [19]. The whole process is that many tech firms drew down from Silicon Valley Bank for operation due to their huge cash consumption during this period. Such fast speed made the bank insolvent, being forced to sell bonds at a loss, which are supposed to be profitable in the future. The run on the bank later quickly cased panic and more depositors came to withdraw cash and finally led

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to domino effect. Going back to DCF approach, we can find out that Silicon Valley Bank's cash flow is normal and healthy, but market sentiments gradually cause its collapse. Consequently, we combine the DCF approach and Black-Scholes Model to combine their advantages to better value internet companies under uncertainty.

According to Gupta and Chevalier's structure [20], the value of the company is the sum of the base value of the company and the value of the real options. The base value of the firm uses DCF approach and value of real options uses Black-Scholes Model. The below equations and their variables are for base value and real option value separately:

Base value:

$$\text{V=Value of firm} = \sum_{t=1}^{n} \frac{FCFFt}{(1+WACC)^t} + \frac{CV}{(1+WACC)^N}$$

- FCFFt: free cash flow
- WACC: weighted average cost of capital
- t: the time period
- N: time period to maturity or exist, also considered as horizon period
- CV: continuing value

Real option value:

$$\text{d1} = In\left(\frac{S}{X}\right) + \frac{(r-\delta+\sigma^2)T}{\sigma T^{0.5}}$$

$$\text{d2} = In\left(\frac{S}{X}\right) + \frac{(r-\delta+\sigma^2)T}{\sigma T^{0.5}}$$

$$\text{C} = Se^{-\delta T}N(d1) - Xe^{-rT}N(d2)$$

- S: expected present value of the cash flows
- X: expected present value of all fixed costs
- T: the period that investment can be efficiently executed
- σ: standard deviation of the growth rate of the cash flows from the investment
- r: risk-free interest rate

Finally, given V and C, value of online brokerage company is V+C and is the value of uncertainty at the time of valuation. This valuation is logically reasonable: it includes both stability gain and risky gain. For the stable profit, it relies on fixed management of the original assets and fits DCF method well; for profit brought by potential investment opportunities, this risky part can be quantified by real option method. In terms of uncertainty of internet companies, their new strategies, projects or investment are usually unpredictable, having potential high return and risk. Real option approach ideally captures uncertainty and makes up for this shortcoming. Nevertheless, the limitation of this approach is it requires a deep understanding of the firm itself to estimate parameters in the equations accurately. The reality is that sometimes we can't know very inside information and tend to estimate parameters under subjectivity in terms of changing environments and company variable strategies. Despite the possible subjective bias, this approach provides a different new perspective for investors about how to value internet companies.

4.3. Fuzzy real option approach based on Black-Scholes Model

The first two methods above use probability theory to access uncertainty, there are, however, other ways than probability to treat uncertainty, or imprecision in future estimates, namely, fuzzy logic and fuzzy sets. In fact, the use of fuzzy sets originated from Zadeh in 1978 and he claimed" intrinsic in natural languages is mainly possibilistic rather than probabilistic in nature" to show how fuzzy is more important [21]. From the view of mathematics, this claim may not be proved directly, but it brings us another perspective to value uncertainty by using fuzzy sets rather than probability. Actually, after Zadeh's paper, many financial researchers or analysts began to apply fuzzy in the financial modeling and there are mainly two ways. The first focuses on using fuzzy measure in contrast with a probability measure where a sub-additive property is not fulfilled and the second is to introduce fuzzy parameter such as volatility and interest rates to the price model [22]. We take one fuzzy real option model by Stoklasa and Collan [23] and modify it to see its availability.

We still use the framework of the second approach in 4.2, that is, value of the internet company is the sum of present value and real option value. But we use fuzzy sets to adjust it and here are their equations.

For fuzzy present value:

$$\widetilde{CF} = [\ CF - (1-\gamma)\alpha,\ CF + (1-\gamma)\beta\ ]$$

$$\mathrm{PV} = \sum_{t=1}^{t=n1} \frac{\widetilde{CF}}{(1+WACC)^t} + \sum_{t=n1+1}^{n} \frac{\widetilde{CF}}{(1+WACC)}$$

Here, the left width α is an important parameter to measure the negative value range of fuzzy number and right width β is to measure the forward value range of fuzzy number on the other hand. Finally we get PV=[PV1, PV2], PV1 and PV2 are calculated according to CF1 and CF2 separately. PV1 is the conservative estimation and PV2 is the optimistic estimation, which creates an interval to make valuation more comprehensive.

For real option value:

$$\tilde{S} = [S - (1-\gamma)\alpha, S + (1-\gamma)\beta]$$

$$\widetilde{K} = [K - (1-\gamma)\alpha, K + (1-\gamma)\beta]$$

Here, confidence level γ is an important index to measure the fuzzy degree of parameters.

Given $\tilde{S}$ and $\widetilde{K}$, we can calculate adjusted d1 and d2, and eventually C according to Black-Scholes Model. Similarly, we have two C here, one conservative and another optimistic. In sum, we add present value and real option value to get the total value of the internet company

And the formula is total value = [PV1+C1, PV2+C2].

Because of this approach's interval setting, we can use it to make the decision more comprehensively combined with fundamental analysis. Undoubtedly, when PV1+C1 >0, this internet company can be considered as good or have great potential in most cases because even the most conservative estimation is positive. Further, if the most conservative estimation is negative, we need to check PV1+C2 and PV2+C1. If both of them are less than 0, we don't consider investment. Otherwise, we set two corresponding situations to do further analysis.

Situation 1: PV1+C2>0, P1+C1<0, holding the conservative valuation of net present value of the internet company, we need to combine with the fundamental analysis to decide whether the company has a profitable future. If so, the total value is positive and it worths investment.

Situation 2: PV1+C1<0, PV2+C1>0, on the other hand, our estimation about the company's future is conservative and the status of the present value of the company is important here. If we are optimistic about net present value of the company, then its total value is positive and worth investment.

However, even though we have tried our best to value randomness and fuzziness in this approach, it is still possible to misuse it if we have serious wrong market analysis. To be specific, if we misunderstand local government's law or misjudge the development of technology in the future, probably we will choose wrong the C value in the interval and make the wrong decision. In fact, there is no completely objective methodology to value accurately. Fuzzy real option approach has already optimized the pure real option and made valuation in a reasonable range.

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5. Conclusion

Because of the internet's rapid development and high uncertainty, valuing internet companies is very challenging. Certainly, there is no single model or method that can predict all types of internet companies. Therefore, corresponding to different kinds of internet companies, different approaches are supposed to be applied to value them more precisely. In this study, three approaches based on the Black-Scholes model are illustrated in depth to show how they value internet companies correspondingly. Metcalfe's approach focuses on valuing internet companies with social media as their main revenue and the prerequisite is that the company has enough reachable potential users. DCF approach and Fuzzy real option approach are mainly for long time evaluation of internet companies, emphasizing the present value and future of the company. The difference is that DCF approach is based on probability and Fuzzy real option is more based on Fuzzy theory.

These three methods provide a new perspective of observation and research ideas, improving the traditional valuation methods, and establishing a useful analytical framework for the valuation of internet companies. Definitely, in addition to its great significance in theory and practice, the limitations still exist since fundamental analysis is still required to estimate part of parameters in approaches, inevitably bringing possible deviation from the more accurate valuation. Meanwhile, the valuation of internet companies has many other possible factors and it's tough to take all of them into account. Hence, facing diversification of internet companies and increasing uncertainty, researchers and analysts can try to further quantify unpredictable situations and put forward new methods for new types of internet companies.

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