

An Analysis of the Fama-French Three-Factor Model's Capacity to Account for Cross-sectional Volatility in Stock Returns

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Abstract: The study's main focus is on the Fama-French Three-Factor Model's capacity to account for cross-sectional volatility in stock returns. The Fama-French Three-Factor Model was used to assess a sample of 720 equity funds and determine each fund's excess returns and three-factor risk exposures. The exposure of each element was then tested using linear regression to see if it could predict the excess returns. The study's focus is on the shortcomings of the traditional Capital Asset Pricing Model (CAPM) and the possibility that the 3-factor model could offer a more precise and thorough explanation of stock returns. The empirical results indicate that it may not be robust all the time, and there could be other factors functioning meanwhile, which are not captured by the model. Furthermore, the Efficient Market Hypothesis (EMH) may not be effective and accurate in its strictest form. The study's empirical examination of the Three-Factor Model and its implications for the Efficient Market Hypothesis (EMH) and other related ideas are what give it its value.

Keywords: The Fama-French Three-Factor Model, EMH, size factor (SMB), value factor (HML)

1. Introduction

The relationship between risk and expected returns is outlined in CAPM, which is also a widely used financial model. Fama's Three-Factor Model, represents an extended one of CAPM with additional factors considered, which determine the returns brought by an asset. The Fama-French model plays a crucial role in finance as it presents a comprehensive picture of the factors that have their effect on the returns produced by an asset, and enables investors to make informed and rational decisions on investment. EMH is a fundamental assumption in finance. It asserts that financial markets are effective. Meanwhile, asset prices can reflect all available information [1]. According to the hypothesis, it is impossible to overturn the market by using the already publicly available information all the time, as the new information is implied quickly and accurately in the asset prices. Generally, both Fama's Three-Factor Model and CAPM are related to the EMH, as both of them are based on the assumption that asset prices reflect all available information. Meanwhile, the former one includes additional factors beyond market risk, while the latter focuses primarily on market risk. The research topic is the Three-Factor Model and its ability to explain cross-sectional changes in stock returns. Based on this topic, the author analyzed a sample of 720 equity funds and used a model to calculate

the excess return and three factor risk exposure of each fund. Then linear regression can be used to examine whether the exposure of each factor could predict the excess returns. The value of the study lies in its empirical analysis of the 3-factor model and its implications for EMH and other related concepts.

2. Literature Review

Sharpe gave a general introduction of the CAPM in his paper [2]. In his research, he also assumed that investors hold a diversified portfolio of assets. The risk of a single asset is measured by its beta coefficient. This coefficient reflects the sensitivity of assets to market risk. The article presents the CAPM equation, which states that the expected return of an asset equals the risk-free rate plus a premium proportionate to the asset's beta [3]. Lintner further developed Sharpe's CAPM by introducing investor preferences in his paper [4]. It is assumed that investors have a utility function to incorporate risk aversion and meanwhile people can choose their portfolio based on their own preferences. Lintner also introduced the idea of dividend policy, and how it affects the expected return of an asset.

Fama and French present the Three-Factor Model [5]. They come to the conclusion that the model, as opposed to the CAPM, provides a superior explanation for the cross-sectional variation in returns. Moreover, they propose a new asset pricing model with five factors covered, including market risk, size, value, profitability, and investment [6]. They believe that these five factors perform better in explaining the return on stock investment portfolios. This performs better than the CAPM or Fama French Three-Factor Model. Also, they offered practical evidence to support the validity of their five-factor model.

Eugene Fama and Kenneth French updated their research by using their five-factor model to study various stock market anomalies [7]. They discovered that this model can explain the returns on many of these anomalies, including the momentum anomaly, the profitability anomaly, etc. The authors argue that these anomalies can be explained by these five factors shown in their asset pricing model. Additionally, they also provide evidence that the model can explain the financial acts more efficiently and effectively than the Fama-French Three-Factor Model for these anomalies. This research tests relationships between beta and alpha by 3-factor model based on existing literature. Moreover, the conclusion that whether there is significant relationship between beta and alpha is meaningful; hence, EMH was tested.

3. Methodology

3.1. Model

The CAPM is mainly available for the pricing of stocks. According to the assumptions of the CAPM, investors can borrow and lend risk-free (r_f) funds. The formula for the CAPM model is

$$E(R_i) = r_f + \beta_i * [E(R_m) - r_f] \quad (1)$$

where $E(R_i)$ denotes the expected return rate of a stock, r_f means the risk-free return rate, β_i denotes the beta value of the stock, and $E(R_m)$ illustrates the average return rate of all stocks. The reason for increasing the risk-free return rate is that the expected return rate must be greater than it. The excess return rate alpha is defined as $E(R_m) - r_f$, which subtracts the risk-free return rate from the market's average return rate for all stocks [8]. If the value of alpha is negative, it means that investing in stocks is not as good as fixed-term deposits. Investors naturally hope that alpha will be higher, which represents a higher return.

CAPM has been faced with some criticism due to its simplicity and its inability to fully explain the cross-sectional variation in stock returns. The 3-factor model supplements two additional factors to CAPM, including SMB and the value factor (HML). SMB explains a trend where small cap stocks outperform large cap stocks. HML shows a trend of value stocks outperforming growth stocks. By incorporating these additional factors, the model provides a more well-developed framework for grasping the cross-sectional variation in stock returns.

The Fama-French Three-Factor Model equation is:

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + s_iSMB + h_iHML + \varepsilon_i \quad (2)$$

Where R_i is the extra return generated by a specific fund; R_f is the risk-free rate of return; α_i is the part of the extra return that cannot be explained by market risk exposure or other factors. It's the intercept or excess return; β_i represents the sensitivity of the fund. This sensitivity is related to overall market changes.

R_m is the overall return of the market. SMB is the difference in returns between small and large companies. This is known as the size factor; s_i is the sensitivity of a fund to differences in returns of small vs. large companies (the size factor); HML measures the variance in returns between growth and value stocks. This is known as the value factor; h_i is the sensitivity of a fund to the differences in returns of value vs. growth stocks (the value factor); ε_i is the remaining extra returns of a fund after the effects of all other factors have been accounted for. This is also known as the error term or unexplained part of the excess return of the fund.

3.2. Samples and Variables

Relevant data comes from JQDATA [9]. There are several samples including all equity funds established before January 1, 2020 and traded continuously on the exchange until January 1, 2023. The net asset value data of 720 funds were collected. These samples were selected on the basis of the continuous operation and trading of funds.

As a result of the impact of pandemic prevention and control on fund managers' ability to generate returns, the net asset value data of 720 funds was collected to maintain a large enough sample number, and therefore avoid low probability events. The market risk is represented by the following three factors, such as market value, and value factors, respectively the CSI 300 Index, the CSI 500 Index ETF, and the SSE 50 Value ETF. Their codes are 399300, 510500, and 510630 in several.

The net asset value of equity funds is a variable that is typically adopted to calculate the fund's return. To calculate the net asset value, this is necessary. Firstly, calculations must be made based on the changes in stock prices held by the fund. The stock price is usually calculated based on the closing price of the day. As a result, the return of equity funds can be calculated by dividing the change in the fund's net asset value between yesterday and today by yesterday's net asset value and multiplying by 100%. The calculation formula is:

$$\text{Return of Equity Funds} = \frac{(\text{Net Asset Value} - \text{Previous Net Asset Value})}{\text{Previous Net Asset Value}} \times 100\% \quad (3)$$

Both of the dependent and independent variables of the model come from the market return, market value, and book-to-market ratio factor analyses of company financial data.

4. Empirical Results

In terms of the 720 equity funds in the sample, the model was applied to obtain the excess returns(α_i) and three-factor risk exposure of the equity funds (β_i, s_i, h_i). For the 720 cross-sectional data of excess

returns(α_i) and three-factor risk exposure, a linear regression model was adopted to examine whether the exposure of each factor of each equity fund (β_i, s_i, h_i) can predict the excess returns of the fund(α_i).

OLS estimation of each regression is as following:

$$\beta_i = 0.0320 + 46.9870\alpha_i \quad (4)$$

$$p - \text{value } (0.000)(0.013) \quad (5)$$

The datas of F-statistic and the probability are 6.171 and 0.0132, respectively, which indicate that, at a 5% level of significance, the model is statistically significant. Besides, The p-value of α_i is not greater than 0.05, which indicates that the coefficient is statistically important.

$$s_i = 0.5020 + 15.7583\alpha_i \quad (6)$$

$$p - \text{value } (0.000)(0.788) \quad (7)$$

The model's F-statistic and p-value demonstrate that the model as a whole is not statistically significant enough. The p-value for α_i is more than 0.05 indicating that the coefficient is not statistically necessary.

$$h_i = 0.3855 - 110.1291\alpha_i \quad (8)$$

$$p - \text{value } (0.000)(0.040) \quad (9)$$

The F-statistic is 4.230, and its p-value is 0.0401, and these two values shows that the model as a whole is statistically important at a significance level of 0.05. The p-value of α_i is not greater than 0.05, which indicates that the coefficient is statistically crucial.

5. Discussion

It is indicated that the sensitivity of the fund to the market factor ($R_m - R_f$) and its sensitivity to the HML factor can be explained or predicted by excess return unexplained by market risk exposure or the Fama-French factors, instead of its sensitivity to the SMB factor. The study mentioned previously investigates whether the 3-factor model is absolutely robust all the time or whether there are some other factors that could be relevant to account for cross-sectional variation in returns. The results illustrates that the excess return of the fund can forecast its sensitivity to the market factor and the HML factor rather than the SMB. This means that the 3-factor model may not always be robust, and other factors could also function effectively in explaining expected stock returns. In addition, it can be concluded that the 3-factor model partially explains the cross-sectional variation in returns, indicating the Efficient Market Hypothesis (EMH) may not stay in its strictest form all the time. EMH states that all available information about a stock is obviously reflected in its price [10]. Therefore, it is impossible for investment to consistently outperform the market over time. However, it is suggested in the model that there exist some other factors investors can consider to gain an edge and earn excess returns. The results, however, may contradict what is predicted in EMH.

However, there are limitations to this empirical method. The assumption of this method is that the model is the most suitable model. It explains the change of cross section of returns of all stock fund in the relevant sample [11]. However, there may be other factors that are relevant for certain subsets of the funds but are not noticed or discussed by this model. Furthermore, the samples here may not represent the entire population of equity funds, which could lead to biased or inaccurate results. Another limitation is that the linear regression model is able to show a linear relationship between the

excess returns and the three-factor risk exposure of each equity fund in advance. This may not always be the case, however, and the relationship could be non-linear or even nonexistent. This limitation could result in a poor fit of the model as well as inaccurate predictions of excess returns. Furthermore, the potential influence of other variables is not considered here, such as liquidity, momentum, and volatility, which may also affect stock returns. Neglecting these variables could lead to incomplete or biased results. Finally, the method relies on historical data to estimate factor exposures and predict future returns.

6. Conclusion

The Empirical Results indicate that Fama and French Three-Factor Model may not be robust all the time and there could be other factors functioning meanwhile, which are not captured by the model. Furthermore, EMH may not be effective and accurate in its strictest form. The Three-Factor Model is further improved upon the CAPM by combining extra factors. These additional factors can to some extent explain the cross-sectional changes in stock returns. Future studies could focus on analyzing the performance of other multi-factor models. Also, they can compare their ability to explain the cross-sectional variation in returns. Additionally, researchers could attach their attention to investigating the role of other variables that may affect stock returns, such as liquidity, momentum, and volatility. These studies would provide a more comprehensive understanding of the factors that drive stock returns and help determine whether EMH maintains its strictest form. Furthermore, studies could also explore the use of alternative statistical techniques, such as machine learning algorithms, so as to quickly know the complexity of stock market dynamics and get more accurate predictions of returns.

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