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A Study of Asian Chooser Options in a General Two-Period Binomial Model

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Abstract: Asian options are also known as average price options, and their strike price is the average price of the stock in the market half a day before the strike. Binomial model is a good model to identify the value of options, The paper develops the theory of Asian chooser options in the two-period binomial model. There are Asian call options, and Asian put options. We first derive the binomial model of the Asian option and get the portfolio function. After that, we get the number of bond and stock, Nb and Ns. We study the problem via five different cases in the two-period Binomial model. In these five cases, we choose the maximum value between Asian put and call options. Then, we decide whether they are call options or put options. Then, we calculate the number of portfolios in these five cases in the two-period binomial model. Finally, we implemented the whole project with Python.

Keywords: Asian options, two-period binomial model, stock

1. Introduction

The two-period binomial model is used to describe the theory of the Asian chooser option. The Binomial model is a good model to identify the value of options. It does not need too much of mathematics, so it's easy to understand.

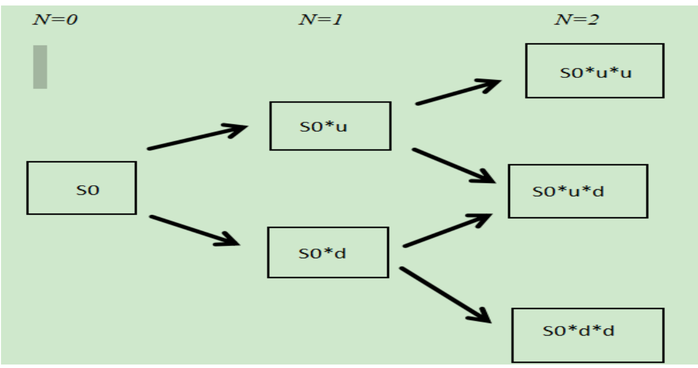

Figure 1: Two period binomial model

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There are five variants of the pricing for Asian options in the binomial model(As the figure 1 shows). K is the strike price, which is the price of the transaction. Moreover, u means the raw return if the stock goes up and d is the raw return if the stock price goes down. Finally, S0 is the initial price of the stock. Nb means the number of bonds, Ns means the number of stocks. Finally, r is the interest rate [1].

A standard option is a financial contract that gives its holder the right to sell or buy the asset at a certain time *and* price. If the holder can only exercise the option at the expiration of the contract, then this option contract is a European option; if investors can exercise their rights at any time before the expiration date of the contract, it is an American option.

Asian options are also known as average price options. It is an exotic option. Its execution price is the average *price* of the market stock prices in the first half of the execution day. The proceeds of Asian options depend on the average price of the asset over a period during its validity period Asian options can be divided into two kinds, the fixed strike Asian option and the floating strike option. The average price options are cheaper than standard options with lower option risk and time value [2,3]. The revenue of an average exercise price option is the difference between the spot exchange rate at the time of exercise and the average price of the underlying asset. This option can ensure that the average price paid for purchasing assets that are frequently traded within a certain period of time is lower than the final price. In addition, it can also ensure that the average price obtained by selling assets that are frequently traded over a period of time is higher than the final price [4,5].

1.1. Our Contribution

Our contribution is to use two-period Binomial to demonstrate the Asian chooser options. During this process, we use Python to get the results of five different cases. Specifically, they choose the maximum value between the call options and put options.

1.2. Paper Structure

The rest of the paper is organized as follows. Section 2 introduces the mathematics of Asian chooser options, which include calculating payoff input options and call options, Arithmetic Asian Options in 5 Cases. The third part is the simulation. We use Python to program the code to get the results of the Asian chooser option. Finally, the fourth part is the conclusion.

2. Background

2.1. Price an Asian Option

There are two primary formulas to express how to calculate payoff in call options and put options respectively. $Payoff^c = max\left[\left(\frac{S1+S2}{2}\right) - K, 0\right]$; $Payoff^p = max\left[K - \left(\frac{S1+S2}{2}\right), 0\right]$. Describe the payoff of the option by the function F. Then if the stock goes up, the holder receives $F(S_0u)$, and if the stock goes down, the holder receives $F(S_0d)$. According to the theory that the option price equals to

$$X_0 = N^B \times 1 + N^S \times S_0$$

Hence,

$$X_1 = \begin{cases} N^B(1+r) + N^S S_0 u = F(S_0 u) & (I) \\ N^B(1+r) + N^S S_0 d = F(S_0 d) & (II) \end{cases}$$

Then N^B and N^S can be expressed by two different formulas.
Taking the difference between equation (I) and (II) to get the formula of N^S

$$N^S = \frac{F(S_0u) - f(S_0d)}{S_0(u-d)} \quad ①$$

Taking the difference between $d \times (I)$ and $u \times (II)$ to get the formula of N^B

$$N^B = \frac{1}{R}\frac{dF(S_0u) - uF(S_0d)}{d-u} \quad ②$$

Hence, N^B and N^S in X_0 can be $replaced$ by $formula$ ① and ②, then get

$$X_0 = \frac{1}{R}\left[F(S_0u)\frac{R-d}{u-d} + F(S_0d)\frac{u-R}{u-d}\right]$$

We define that $q = \frac{R-d}{u-d}$. From above, we can get

$$X_1^C(u) = \frac{1}{R}\left[q \times max\left[\left(\frac{S_0u^2+S_0u}{2} - K\right), 0\right] + (1-q)max\left[\left(\frac{S_0ud+S_0u}{2} - K\right), 0\right]\right];$$

$$X_1^C(d) = \frac{1}{R}\left[q \times max\left[\left(\frac{S_0ud+S_0d}{2} - K\right), 0\right] + (1-q)max\left[\left(\frac{S_0d^2+S_0d}{2} - K\right), 0\right]\right]$$

so we infer the option price in call options can be expressed as

$$X_0^C = \frac{1}{R^2}[qX_1(u) + (1-q)X_1(d)]$$

Similarly, we get

$$X_1^p(u) = \frac{1}{R}\left[q \times max\left[\left(K - \frac{S_0u^2+S_0u}{2}\right), 0\right] + (1-q)max\left[\left(K - \frac{S_0ud+S_0u}{2}\right), 0\right]\right];$$

$$X_1^p(d) = \frac{1}{R}\left[q \times max\left[\left(K - \frac{S_0ud+S_0d}{2}\right), 0\right] + (1-q)max\left[\left(K - \frac{S_0d^2+S_0d}{2}\right), 0\right]\right]$$

And the option price in put options can be expressed as

$$X_0^p = \frac{1}{R^2}[qX_1(u) + (1-q)X_1(d)]$$

2.2. Arithmetic Asian Option in 5 Cases

Considering different range of K contributes to different payoff, we separate the problem into 5 cases.

Case A: $\boldsymbol{K > S_0u\frac{1+u}{2}}$

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The outcomes of payoff in call options are zero, then we found $X_0 = 0$ in call options.

$$X_1^C(u) = 0;\ X_1^C(d) = 0$$

In put options, outcomes of payoff are positive

$$X_1^p(u) = \frac{1}{R}\left(K - \frac{R+1}{2} S_0 u\right);$$

$$`X_1^p(d) = \frac{1}{R}\left(K - \frac{R+1}{2} S_0 d\right);$$

The most important step is to choose the bigger one of X_1 if stock goes up or goes down. In this case, $X_1^p(u)$ is bigger if stock goes up and $X_1^p(d)$ is bigger if stock goes down. Then we can compute the price with $X_1^p(u)$ and $X_1^p(d)$.

$$X_0 = \frac{1}{R^2}(K - \frac{R(R+1)}{2} S_0)$$

Case B: $\boldsymbol{S_0 u \frac{1+d}{2} < K < S_0 u \frac{1+u}{2}}$

In call options, if stock goes up, the outcome of payoff is positive. But if stock goes down, the outcome of payoff is zero, so

$$X_1^C(u) = \frac{R-d}{R(u-d)}(S_0 u \frac{1+u}{2} - K);\ X_1^C(d) = 0$$

In put options, outcomes of payoff are positive

$$X_1^p(u) = \frac{u-R}{R(u-d)}(K - S_0 u \frac{1+d}{2});$$

$$X_1^p(d) = \frac{1}{R}\left(K - \frac{R+1}{2} S_0 d\right)$$

So we need to compare $X_1^C(u)$ with $X_1^p(u)$, and we compute the difference of two formulas.

$$X_1^C(u) - X_1^p(u) = S_0 u \frac{1+R}{2R} - \frac{K}{R}$$

Hence, we divided it into two cases,

Case $\boldsymbol{B_1}$: $S_0 u \frac{1+R}{2} > K$

We get $X_1^C(u) = \frac{R-d}{R(u-d)}(S_0 u \frac{1+u}{2} - K)$ and $X_1^p(d) = \frac{1}{R}\left(K - \frac{R+1}{2} S_0 d\right)$. So we can compute the price in this case.

$$X_0 = \frac{1}{R^2}[\frac{(R-d)^2}{(u-d)^2}\left(S_0 u \frac{1+u}{2} - K\right) + \frac{u-R}{u-d}\left(K - \frac{1+R}{2} S_0 d\right)]$$

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Case B_2: $S_0 u \frac{1+R}{2} < K$

Considering the precondition of Case B: $\boldsymbol{S_0 u \frac{1+d}{2} < K < S_0 u \frac{1+u}{2}}$ and R must be bigger than d, so $\frac{1+R}{2} > \frac{1+d}{2}$ so it is impossible to reach $S_0 u \frac{1+R}{2} < K$. Therefore, we can except **Case B_2.**

Case C: $\boldsymbol{S_0 d \frac{1+u}{2} < K < S_0 u \frac{1+d}{2}}$

In call options, if stock goes up, the outcome of payoff is positive. But if stock goes down, the outcome of payoff is zero, so

$$X_1^c(u) = \frac{1}{R}\left(\frac{R+1}{2} S_0 u - K\right);\ X_1^c(d) = 0$$

In put options, if stock goes up, the outcome of payoff is zero. But if stock goes down, the outcome of payoff is positive, so

$$X_1^p(u) = 0;\ X_1^p(d) = \frac{1}{R}\left(K - \frac{R+1}{2} S_0 d\right)$$

So, we can choose call options if stock goes up and choose put options if stock goes down and compute the price

$$X_0 = \frac{1}{R^2}[K + \frac{(R^2-1)(u-d)}{2(u-d)} S_0$$

Case D: $\boldsymbol{S_0 d \frac{1+d}{2} < K < S_0 d \frac{1+u}{2}}$

In call options, the outcomes of payoff are positive

$$X_1^c(u) = \frac{1}{R}\left(\frac{R+1}{2} S_0 u - K\right);$$

$$X_1^c(d) = \frac{R-d}{R(u-d)}(S_0 d \frac{1+u}{2} - K)$$

In put options, if stock goes up, the outcome of payoff is zero. But if stock goes down, the outcome of payoff is positive, so

$$X_1^p(u) = 0;\ X_1^p(d) = \frac{u-R}{R(u-d)}(K - S_0 d \frac{1+d}{2})$$

From above, if stock goes up, $X_1(u) = \frac{1}{R}\left(\frac{R+1}{2} S_0 u - K\right)$; if stock goes down, we need to compare $X_1^c(d)$ with $X_1^p(d)$. Computing the difference of $X_1^c(d)$ and $X_1^p(d)$. $X_1^c(d) - X_1^p(d) = S_0 d \frac{1+R}{2R} - \frac{K}{R}$ then we divided it into two cases.

Case D_1: $\boldsymbol{S_0 d \frac{1+R}{2} > K}$

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Considering the precondition of Case D: $\boldsymbol{S_0 d \frac{1+d}{2} < K < S_0 d \frac{1+u}{2}}$ and R must be smaller than u, so $\frac{1+R}{2} < \frac{1+u}{2}$ so it is impossible to reach $S_0 d \frac{1+R}{2} > K$. Therefore, we can except **Case $\boldsymbol{D_1}$.**

Case $\boldsymbol{D_2}$: $\boldsymbol{S_0 d \frac{1+R}{2} < K}$

We get $X_1(u) = \frac{1}{R}\left(\frac{R+1}{2} S_0 u - K\right)$ and $X_1(d) = \frac{u-R}{R(u-d)}(K - S_0 d \frac{1+d}{2})$. So we can compute the price in this case.

$$X_0 = \frac{1}{R^2}[\frac{R-d}{u-d}\left(S_0 u \frac{1+R}{2} - K\right) + (\frac{u-R}{u-d})^2 \left(K - \frac{1+d}{2} S_0 d\right)]$$

Case E: $\boldsymbol{K < S_0 d \frac{1+d}{2}}$

In call options, outcomes of payoff are positive

$$X_1^c(u) = \frac{1}{R}\left(\frac{R+1}{2} S_0 u - K\right);$$

$$X_1^c(d) = \frac{1}{R}\left(\frac{R+1}{2} S_0 d - K\right);$$

The outcomes of payoff in put options are zero, then we found $X_0 = 0$ in put options.

$$X_1^p(u) = 0;\ X_1^p(d) = 0$$

In this case, $X_1^c(u)$ is bigger if stock goes up and $X_1^c(d)$ is bigger if stock goes down. Then we can compute the price with $X_1^c(u)$ and $X_1^c(d)$.

$$X_0 = \frac{1}{R^2}(\frac{R(R+1)}{2} S_0 - K)$$

3. Simulation

In order to show the results of Asian chooser option more directly, we decide to use the Python to program the code of it. Figure 2 shows the code of program.

```
u=2
d=1/2
S0=100
k=30
R=1
q=(R-d)/(u-d)

if k<S0*(d+d*d)/2:
    x1u = 1/R*S0*u*(R+1)/2-k/R
    x1d = 1/R*S0*d*(R+1)/2-k/R
elif k<S0*d*(u+1)/2:
    x1u=1/R*S0*u*(R+1)/2-k/R
    x1d=max((1/R)*(S0*d*(1+u)/2-k)*q,(1/R)*(S0*d*(1+d)/2-k)*(1-q))
elif k<S0*u*(1+d)/2:
    x1u=1/R*S0*u*(R+1)/2-k/R
    x1d=1/R*(k-S0*d*(R+1)/2)
elif k<S0*u*(1+u)/2:
    x1u=max((1/R)*(S0*u*(1+u)/2-k)*q,(1/R)*(k-S0*u*(1+d)/2)*(1-q))
    x1d=1/R*(k-S0*d*(R+1)/2)
else:
    x1u=1/R*(k-S0*u*(R+1)/2)
    x1d=1/R*(k-S0*d*(R+1)/2)
x0=1/R*(x1u*q + x1d*(1-q))
print(x0)
```

Figure 2: Code

Therefore, there are five variables above(u,d,S0,k,R,q).

So, we can change the magnitudes of these variables in different situations, then code will allocate different situations into different steps. If the situation satisfies the first condition k<S0*(d+d*d)/2 (Case E), then it will be still judged whether it meets the second condition, if not, the price of Asian chooser option will be calculated in the first condition, but if it still meets the second condition, then it will be judged in the third condition, and so on. Therefore, the price of Asian chooser option can be calculated by this code in every different situation.

4. Conclusion

The Asian chooser option has one thing common with European option, and that are they only allowed to execute their option at expiration date, but the difference is that European options are based on the price of the expiration date S(T), Asian options are based on the average price of the stock price within the contract period [0, T] to decide whether to execute the option contract. Therefore, the use of average price of stock price may reduce uncertainty, so maybe the Asian chooser option may be more cheaper than other options, so the reduction of cost may let them become more popular.

It can be used to protect against both exchange rate and interest rate of risks related to regular or irregular cash flows. For example, a multinational company regularly receives foreign currency cash inflows, (which are either dividends, dividends or concession transfer fees from subsidiaries and payments from the parent company selling goods overseas,) In order to ensure that these revenues are converted into a country's currency at a specific average exchange rate, the company can purchase a average foreign currency put option. Its effect of reducing foreign exchange risk exposure is the same as purchasing a series of standard European-style put options, but the use of average options can result in significant savings in option costs.

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