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# A Review of the Option Pricing Model and Further Development

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***Abstract:*** The Black Scholes model and binomial tree model have been the main research objects of scholars in the past fifty years. This article summarizes the optimization process of these two classic option pricing models to understand the different directions of optimizing the models and to provide ideas for future model improvement. Improvements from scholars to the Black-Scholes model mainly focus on the basic model and the relevant variables involved in option pricing, while the optimization of the binomial tree model focuses on the reduction of pricing errors as well as the improvement of model fitting speed. Empirical studies of option pricing models have shown that improved models while enhancing the accuracy of pricing in various financial markets, unavoidably increase computational complexity and reduce efficiency. This article also demonstrates the future integration of financial derivatives pricing models in multiple fields by describing the application of real options in the agriculture, technology, and biology industries.

***Keywords:*** Black-Scholes model, Binomial tree model, Real option

## 1. Introduction

As a financial derivative, an option is the right to buy or sell a specific underlying asset at a predetermined price (the strike price) on or before a specified expiration date. The option valuation model is a crucial part of derivatives trading and has always been the focus of scholars. Option pricing models excel in accurately replicating the ideal pricing of corporate debt. Simultaneously, these models are effective tools for evaluating the risk associated with loan guarantees and pension insurance. Currently, quantitative evaluation techniques rooted in option pricing models are utilized for shaping insurance compensation guidelines, conducting flexible capital budgeting analyses, and quantifying the level of risk within product portfolios. At this stage, the most commonly used option pricing model is the Black-Scholes model, which is aimed at solving partial differential equations. Meanwhile, the binomial tree model is also widely utilized in dynamic coding. In this paper, we review the relevant literature on option pricing models, discuss the parameter improvement and innovative development of pricing models from the perspectives of theoretical analysis and empirical testing, and explore the further development of option pricing models in the field of real options valuation.

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## 2. Theoretical development

### 2.1. Black-Scholes Model

In 1973, Black and Scholes analyzed a large amount of data and found that the fluctuations in derivatives markets and underlying assets conformed to Brownian motion with a drift term[1]. Building upon this observation, they applied the no-arbitrage theory to formulate a differential equation expressing the price of the European option consisting of stock prices and expiration time. Consequently, they obtained the analytical solution of that differential equation and derived the specific formula.

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad (1)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \quad (2)$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \quad (3)$$

In the formula(1) to (3) , C is the option value; S is the spot price of the stocks; r is the interest rate that is risk-free; K is the strike price at which options buyer exercises the option; and N (-) denotes the probability function of the standard normal distribution. The Black-Scholes model provides a theoretical framework for estimating the fair market value of options and has been influential in the development of financial markets. However, certain simplifying assumptions have been made, comprising constant volatility and a constant risk-free interest rate, which may not always hold in real-world situations.

#### 2.1.1. Directions For Improvement: Expression of the model

The model assumes that the stock fluctuation is less volatile, and when underlying assets have high volatility, the final value obtained by the Black-Scholes formula will have larger errors compared to the actual market price. To reduce the restrictions of the model, financial researchers have combined stochastic calculus into that model to satisfy Poisson distribution. They argue that the volatility of option prices should be a continuous process, a sum of processes with discrete jumps. Therefore, a jump-diffusion model is proposed, and the pricing formula under this model is derived by introducing the Fourier transform. It also verifies the effectiveness of the jump-diffusion model when the price of the underlying stocks rises or falls sharply.

Kou discovered that some types of financial derivatives are different in comparison with the discrete jump process in the traditional way [2]. Some scholars construct double exponential jump-diffusion models to explain the situation when the underlying assets undergo multi-level jump changes.

In order to reflect actual option market prices more accurately, Bollerslev introduced the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model [3]. In terms of the continuous optimization of the volatility of the underlying stock price, the model combines the volatility as a variable with the stochastic calculus of discrete changing processes. Compared with the 1973 Black Scholes model, the GARCH model is more direct to use and has greater practical use value. Menn and Rachev introduced residuals for underlying stocks to analyze the dynamic effect of GARCH option pricing models [4]. Kim and Rachev propose a slow-increasing stable distribution GARCH model to address the fact that the actual volatility of the option return differs greatly from the simulation results of the traditional GARCH option pricing model [5]. The GARCH option pricing

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model allows for a more realistic representation of option prices in the presence of changing volatility patterns. It's particularly useful for pricing options on assets with time-varying volatility, such as stock market indices or individual stocks. It helps capture the fact that implied and historical volatilities tend to fluctuate over time, especially in response to events and market conditions. However, implementing the GARCH option pricing model can be computationally intensive due to the need for simulation and estimation of GARCH parameters. Additionally, it may require more historical data and parameter estimation compared to simpler models like the Black-Scholes model. Nevertheless, it provides a valuable tool for accurately pricing options in dynamic market environments.

#### 2.1.2. Directions For Improvement: Pricing Factor

Regarding pricing parameters, the traditional Black-Scholes model makes the assumption that both standard deviation and the risk-free interest rate are constants. However, this assumption falls short of capturing the dynamic price change of the options market. Consequently, both international and domestic scholars have largely replaced these two parameters with time-varying function variables. When it comes to interest rates, earlier researchers often substituted short-term interest rates to create options pricing models with stochastic interest rates. However, as monetary policies evolved in various countries, it became evident that a single interest rate could not accurately mimic the real-world risk-free interest rate. This led to scholars questioning the validity of options pricing models. In 2002, Duffee addressed this issue by analyzing different interest rate structures [6]. He introduced the concept of the Affine model, which involves substituting the interest rate in a risk-free situation with a weighted mean of multiple interest rate data. This innovation significantly decreased the uncertainty associated with options pricing models, enhancing their accuracy and applicability. When it comes to volatility, there are two primary approaches to refining the parameters within option pricing models. Some researchers draw inspiration from the jump-diffusion model, where volatility is considered a stochastic process or a discrete function involving two-level jumps with varying probabilities. They replace the constant volatility in the traditional Black-Scholes model with this more dynamic concept. This led to the development of stochastic volatility (SV) pricing models and stochastic volatility jumps (SVJ) models. These models enable the estimation and prediction of implied volatility, offering a more accurate representation of market conditions. Alternatively, some scholars have taken cues from analyzing interest rate structures to enhance the characterization of volatility. For instance, Grünbichler and Longstaff characterized the standard deviation of stocks by examining structural variables and regressing the root mean square of the price [7]. Peiris and Thavaneswaran conducted research on stochastic volatility and used changes in the dynamic trend value of volatility to capture overall volatility shifts [8]. While improving pricing parameters may introduce greater complexity to the Black-Scholes model, it aligns the assumptions more closely with actual options market pricing. Consequently, pricing models become more realistic and reliable in their predictions, reflecting the dynamics of real-world option prices more accurately.

#### 2.1.3. Binomial Tree Model

In 1976, JC Cox and SA Ross described the movement of stock prices for the first time, evaluated the validity of the value of stock purchase options, and the dynamic analysis of option prices, and then proposed the binomial tree pricing model [9].

The idea of the binomial tree model is as follows: if a portfolio can be constructed from stock and an option based on that stock, such that the portfolio return is certain in the final effective duration, then the current value of the investment portfolio construction cost can be obtained. Given that the portfolio has the same rate of return as in the risk-free case, and the portfolio stock price is already known, the value of the option could be calculated.

$$C = \frac{pC_u + (1-p)C_d}{e^{rT}} \tag{4}$$

In the formula (4) is the option value; p is risk-neutral probability; r is the interest rate in the risk-free condition; Cu and Cd are the up and down situations for the value of the option purchase. It becomes evident from this discussion that the core concept of this model is to estimate the fluctuations in underlying asset prices by breaking down the process into discrete, small-amplitude binary operations. Expanding the single-period model to encompass multiple periods to formulate the pricing model.

The advantage of the binary tree model is that it can be used to price different options, and it can be applied in different fields with strong flexibility. As a result, scholars utilize this model to analyze the valuation problem of idiosyncratic options. However, since this model also needs to satisfy a series of assumptions, the model's prediction value has a large error compared with the real price in the options market, and with the increase of the model's prediction period, the computational volume increases exponentially, and the speed of computation decreases gradually, so scholars mainly improve the standard binomial tree model from the perspectives of reducing the pricing error and increasing the speed of the model fitting.

#### 2.1.4. Directions For Improvement: Making Models More Accurate In Predicting Specific Options

Hull and White evaluated the Bermudian option pricing model by using a control variable approach in 1988, adding the more important time variable, and treating the option price as a function of time, which further improved the predictive reliability of the Bermudian option price[10]. Ritchken used a binomial tree option pricing approach to simulate and predict options with hurdle levels close to the price of the underlying asset and found that the fitted results differed significantly from the true values [11]. Based on this, Figlewski and Gao introduced adaptive lattice structure variables into the binomial tree model for a more refined approximation of the barrier option pricing problem and found that the improved model did manage to yield desirable results in terms of accuracy [12]. Since Figlewski and Gao's improvement makes the computation of the model's results more complicated, the efficiency of the pricing model is substantially reduced. To balance the accuracy and efficiency, scholars modified the binomial tree model by introducing singularities, path interpolation, and other methods to improve the performance of the barrier option pricing model. Boyle and other scholars developed a trinomial basic model based on the binomial tree model based on fuzzy mathematical theory, which aims to improve the jagged bounded options that are difficult to converge and to make the calculation of the barrier options easier [13]. The trinomial tree model is more complex in comparison with the binomial model due to the additional price movement state, but it can provide a more accurate statement of the underlying dynamic movement of asset prices, especially in situations with asymmetric volatility. It is useful for pricing options on assets with multiple sources of uncertainty or when constant volatility assumptions do not hold.

#### 2.1.5. Directions Of Improvement: Improve The Computational Speed Of Special Option Pricing Models

Breen parameterized the standard binomial tree model and proposed an accelerated binomial tree model for evaluating the pricing accuracy of sub-options [14]. However, these improvements are all aimed at the computational speed of a single option and fail to break through the inefficiency of the

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binomial tree model in analyzing the pricing of a combination of options. The binomial tree model itself is unable to find the exact solution of the option price at the intermediate point in time, and when the underlying price of the derivative product has more than three price variables, the sudden increase in the amount of iterative computation causes the computation time of the binomial tree model to surge, which further restricts the accuracy of the model in analyzing the dynamics of the model. To address this problem, Kamrad and Ritchken proposed a trinomial option pricing model through rigorous mathematical derivation, which not only breaks through the assumption of normal distribution of option return data and portrays the characteristics of "sharp peaks" and "long tails" of actual data, but also realizes the dynamic simulation of price and shortens the operation time of the model [15]. Since then, different scholars have compared the traditional binomial tree with the modified trinomial model from various perspectives, such as elasticity, kurtosis, degree of freedom, etc., which makes the trinomial model have a greater improvement in the speed of computation and the degree of adaptability to complex option pricing.

## 3. Empirical Analysis

As the options market has matured, empirical studies on option pricing have grown increasingly comprehensive. These studies mainly involve the refinement of factors in the pricing model and validation of models. Levy and Byun sought to assess the credibility of Black-Scholes model pricing [16]. They derived implied volatility according to the estimated variance within the confidence interval. Corrado and Su conducted a practical analysis of options of S&P 500 stocks [17]. They discovered a negative correlation between the standard deviation of return and changes in stock index levels. Additionally, they estimated and predicted the parameters of stochastic volatility option pricing models, highlighting the practical value of these models. Saurabha and Tiwari addressed the "smiles problem" in traditional option pricing models[18]. They introduced skewness and kurtosis as statistical variables to estimate the prices of currency options. This approach resulted in forecasts closer to market prices. Andrés-Sánchez analyzed option valuation based on the fuzzy Black-Scholes formula [19]. They fitted relevant parameters to actual trading prices of Spanish stock index options and evaluated the model's ability to predict stock index option prices from various perspectives. Bailey and Stulz conducted analysis and practical tests on financial derivatives pricing using interest rates in the stochastic process and stochastic volatility formula [20]. Yung and Zhang performed dynamic analyses and empirical tests on stock index options pricing, comparing the GARCH option pricing model with the traditional Black-Scholes model [21]. They found the GARCH model outperformed the Black-Scholes model in various evaluations. Kim and Lee proposed a model for estimating volatility under no-arbitrage conditions [22]. They empirically analyzed pricing for the sample data, and hedged uncertainty using KOSPI 200 index options, confirming the model's validity. Oliver and Li studied price data of European call options from the perspective of equilibrium interest rates and consumer capital asset pricing [23]. They aimed to predict the jump time of price jumps in the jump-diffusion option pricing model, offering insights into how model parameters affect actual option pricing.

## 4. Further Research

### 4.1. The Application Of Option Pricing Models

#### 4.1.1. Black-Scholes Model And Real Options

The model was derived by Black and Scholes for financial option pricing, and scholars have since applied the model to real asset pricing, which is also the most widely used [1]. Domestic scholars have also studied the application of the model to the valuation of individual assets and overall assets.

Black-Scholes model was used to assess the value of athletes' human capital, reflecting the true value of athletes' human capital and eventually providing a basis for decision-making on athletes' human capital investment; Xu and Zheng used this model to assess the value of the right to use collective land for construction purposes in rural areas based on considerations with the value of single-period income from land cultivation and the value of the future development and used the 2012 data of Sichuan Province as the basis for the assessment of the value of the use right of rural collective operational construction land [24]. In addition, this model was utilized to evaluate the worth of the use right of building land based on the value of single-period income from land cultivation and values of future development.

#### 4.1.2. Binomial Tree Model And Real Options

Some scholars used a binomial tree model to assess the value of vacant land and showed that the choice of the best future building type is decisive for the value of vacant land. Kellogg and Charnes found that many firms in the biotechnology industry have high stock prices despite the fact that their products are still in the early stages of development and do not generate revenues, so they used the binomial tree model to value high-tech firms and found that the early-stage value of high-tech firms is more accurately reflected in the real options approach [25]. The trinomial tree model was used to value patent asset portfolio and concluded that the model can accurately reflect the actual change status of the expected return of the patent portfolio and achieved the purpose of improving the calculation accuracy by increasing the number of states in each sub-period instead of increasing the number of sub-periods, avoiding the computational complexity.

The binomial tree model is applicable to various types of options on values of cross-border mergers and acquisitions of mining companies, which can take it as intuitive and easy to understand, and the changes in the management's decision-making at each stage and the reasons for it can be visualized in a graphic. Compared with the Black-Scholes model, the binomial tree model is more suitable for the evaluation of cross-border projects with long cycles and high price volatility; Liu, Zou, and Chen compared the applicability of the Black-Scholes model and the binomial tree model for evaluations of the value of carbon intangible assets of the enterprise, and they concluded that the carbon intangible assets basically conform to the assumptions of the Black-Scholes model, and the pricing process can neglect the investor's risk appetite, and only need to bring in the related variables to figure out the value of the physical [26]. They concluded that carbon intangible assets basically conform to the assumptions of the Black-Scholes model, while the assumption of the binomial tree model that the stock price has only two possibilities of increasing or decreasing in the future is inconsistent with the form of the actual change of carbon price and is complicated to compute.

## 5. Conclusion

The B-S model was derived by finding the volatility of the derivatives market is consistent with Brownian motion and summarized the relationships with variables. After forty years, the risk-neutral pricing theory was used to establish the binomial tree model. This article summarizes the literature to introduce the basic formulas of these two typical option pricing models and the optimization process of scholars in the later stage. The Black Scholes model is mainly optimized in terms of the basic model and the pricing factors. The Jump-diffusion model, GARCH model can provide more accurate estimations of the value of the option when the price of the underlying asset is volatile. The Stochastic interest rate option pricing model and SV model consider the interest rate and volatility as time-varying functions, which are more in line with the actual market situation. Binomial tree models are mainly optimized by reducing the equivalence spread and increasing the speed of model fitting. The accuracy of the model was improved by introducing adaptive lattice structure variables and other

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computer technologies into the binomial tree model. The trinomial tree model was created to value the option more precisely for underlying stocks with asymmetric volatility. Another aspect that has been optimized is to reduce the fitting time by implementing dynamic simulations of prices. In addition to theoretical developments, this article summarizes empirical studies by scholars on parameter refinement and validation of model effectiveness. Optimized models provide higher accuracy, but require more complex computations, which reduces the efficiency of the model to varying degrees. Therefore, the efficiency of computational equipment and modern option pricing models are closely related. With the development of the options market, the areas in which options are applied are no longer only in the financial industry, and the birth of real options has become the basis for cross-domain. The B-S model can evaluate human capital as well as rural land use rights. The binomial tree model can be intended for evaluating high-tech companies and the trinomial tree model can be used to value patent assets. This shows that the use of options in different markets has become a trend. This is because of the characteristics of the model itself, which calculates future returns and uses various factors to discount the future value into a current value. The discovery of the option valuation formula stems from the combination of finance and physics, and the future development of the model will be a combination of finance and other industries. Scholars can apply their understanding of options to different fields by drawing on the ideas of real options already in existence.

## Author contribution

All the authors. contributed equally, and their names were listed in alphabetical order.

## References


[1] Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. Journal of Political Economy, 81(3), 637–654. https://doi.org/10.1086/260062

[2] Kou, S. G. (2002). A jump-diffusion model for option pricing. Management Science, 48(8), 1086–1101. https://doi.org/10.1287/mnsc.48.8.1086.166

[3] Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307–327. https://doi.org/10.1016/0304-4076(86)90063-1

[4] Menn, C., & Rachev, S. T. (2005). A GARCH option pricing model with α-stable innovations. European Journal of Operational Research, 163(1), 201–209. https://doi.org/10.1016/j.ejor.2004.01.009

[5] Kim, Y. S., Rachev, S. T., Bianchi, M. L., & Fabozzi, F. J. (2008). Financial market models with Lévy processes and time-varying volatility. Journal of Banking & Finance, 32(7), 1363–1378. https://doi.org/10.1016/j.jbankfin.2007.11.004

[6] Duffee, G. R. (2002). Term Premia and interest rate forecasts in affine models. The Journal of Finance, 57(1), 405–443. https://doi.org/10.1111/1540-6261.00426

[7] Grünbichler, A., & Longstaff, F. A. (1996). Valuing futures and options on volatility. Journal of Banking & Finance, 20(6), 985–1001. https://doi.org/10.1016/0378-4266(95)00034-8

[8] Peiris, S., & Thavaneswaran, A. (2007). An introduction to volatility models with indices. Applied Mathematics Letters, 20(2), 177–182. https://doi.org/10.1016/j.aml.2006.04.001

[9] Cox, J. C., & Ross, S. A. (1976). The valuation of options for alternative stochastic processes. Journal of Financial Economics, 3(1–2), 145–166. https://doi.org/10.1016/0304-405x(76)90023-4

[10] Hull, J., & White, A. (1988). The use of the control variate technique in option pricing. The Journal of Financial and Quantitative Analysis, 23(3), 237. https://doi.org/10.2307/2331065

[11] Ritchken, P. (1995). On pricing barrier options. The Journal of Derivatives, 3(2), 19–28. https://doi.org/10.3905/jod.1995.407939

[12] Figlewski, S., & Gao, B. (1999). The adaptive mesh model: A new approach to efficient option pricing. Journal of Financial Economics, 53(3), 313–351. https://doi.org/10.1016/s0304-405x(99)00024-0

[13] Boyle, P. P. (1988). A lattice framework for option pricing with two state variables. The Journal of Financial and Quantitative Analysis, 23(1), 1. https://doi.org/10.2307/2331019

[14] Jin, X. (2011). Valuation of single patent in patent Asset Securitization based on real options. Statistics and decision, 52-55. Doi: CNKI: SUN: TJJC. 0.2011-04-014.

[15] *Kamrad, B., & Ritchken, P. (1991). Multinomial approximating models for options with K state variables. Management Science, 37(12), 1640–1652. https://doi.org/10.1287/mnsc.37.12.1640*
[16] *Levy, H., & Byun, Y. H. (1987). An empirical test of the black-scholes option pricing model and the implied variance: A confidence interval approach. Journal of Accounting, Auditing & Finance, 2(4), 355–369. https://doi.org/10.1177/0148558x8700200403*
[17] *Corrado, C., & Su, T. (1998). An empirical test of the hull-white option pricing model. Journal of Futures Markets, 18(4), 363–378. https://doi.org/10.1002/(sici)1096-9934(199806)18:4<363::aid-fut1>3.0.co;2-k*
[18] *Saurabha, R., & Tiwari, M. (2007). Empirical study of the effect of including skewness and kurtosis in Black Scholes option pricing formula on S&P CNX nifty index options. SSRN Electronic Journal. https://doi.org/10.2139/ssrn.1075583*
[19] *De Andrés-Sánchez, J. (2017). An empirical assestment of Fuzzy Black and Scholes pricing option model in spanish stock option market. Journal of Intelligent & Fuzzy Systems, 33(4), 2509–2521. https://doi.org/10.3233/jifs-17719*
[20] *Bailey, W., & Stulz, R. M. (1989). The pricing of stock index options in a general equilibrium model. The Journal of Financial and Quantitative Analysis, 24(1), 1. https://doi.org/10.2307/2330744*
[21] *Yung, H. H., & Zhang, H. (2003). An empirical investigation of the GARCH option pricing model: Hedging performance. Journal of Futures Markets, 23(12), 1191–1207. https://doi.org/10.1002/fut.10109*
[22] *Kim, N., & Lee, J. (2013). No-arbitrage implied volatility functions: Empirical evidence from KOSPI 200 index options. Journal of Empirical Finance, 21, 36–53. https://doi.org/10.1016/j.jempfin.2012.12.007*
[23] *Li, O. X., & Li, W. (2014). Hedging jump risk, expected returns and risk premia in jump-diffusion economies. Quantitative Finance, 15(5), 873–888. https://doi.org/10.1080/14697688.2014.946439*
[24] *Xu, Z. & Zheng, F. (2015). Real option pricing method of rural collective operational construction. land use right. The rural economy, 36 to 40. Doi: CNKI: SUN: NCJJ. 0.2015-06-007.*
[25] *Kellogg, D., & Charnes, J. M. (2000). Real-options valuation for a biotechnology company. Financial Analysts Journal, 56(3), 76–84. https://doi.org/10.2469/faj.v56.n3.2362*
[26] *Liu, G., Zou, J. & Chen, C. (2004). An option pricing model with multiple jump processes. Number. of economic and technical economic studies, 106-110. The doi: CNKI: SUN: SLJY. 0.2004-04-017.*