

# ***Reliability Analysis of Tree Topology based on Chinese Banking System***

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**Abstract:** A tree topology is a special type of structure in which many connected elements are arranged like branches of a tree, and is often used to organize computers in a company's network, or information in a database. Tree topologies and Chinese banking system are both a highly centered and hierarchical structure, sharing a wide range of analogies. This article mainly studies the market value, under the circumstances of crossholdings, of individual banks in a banking system, as well as the reliability of the banks at different tiers of the supply chain. This paper analyzes how the network propagates the reliability discontinuities caused by faults. A tree topology model is exploited to facilitate the interpretation. In this model, each node has two branches and at each tier of the supply chain there is one leaf node that does not expand further. There are cross holdings within the same tier but not between adjacent tiers. This paper finds out the relationship of one's reliability with the respective position in the financial network. Two variables are taken into consideration: the multisourcing number as well as the complexity amounting to the level in which it is. In a nutshell, the bigger the multisourcing number is, the greater the reliability is. Also, the higher it is in the financial network, the greater the reliability is. In addition, the endogenous configuration of supply network provides a new method for the study of China's banking system.

**Keywords:** Chinese banking system, tree topology, crossholdings, financial networks, networks, cascades, reliability.

## **1. Introduction**

With the deepening of reform and opening up as well as the continuous promotion of economic system reform, China has formed a diversified banking system, with the coexistence of various ownership structures. Globalization is increasing the various financial interdependencies among various organizations - governments, central banks, investment banks, regulators, etc. - which hold shares, debts and other obligations among themselves. On one hand, such interdependencies can conduce to risk shifting and asset accumulation. On the other hand, it can lead to cascading defaults and failures.[1] This kind of system can be characterized by complex networks. Surprisingly, this system embodies a high similarity with tree topology. Previous studies have shown the cascading failures in a fragile system conducted by a relatively trivial shock. For instance, a local shock might precipitate a cascade of failures and have a global impact. The influences of upstream institutions are taken into considerations while the potential hazards transmitted by peer institutions remain

undiscovered. For instance, depositors may withdraw the deposit from other banks in advance in order to pay off the debt or meet the clients' demand, which results in the inability to liquidate or insolvency or failure of other banks.

To begin with, the basic configuration of the Chinese banking system, including the intuition, components, policies, etc. will be elaborated. The scope is then laid on the definition, basic structure, advantages and disadvantages of the tree topology. Through the illustration, this paper summarized their similarities by their main characteristics, through which the feasibility of applying tree topology to the banking system is attained.

This paper develops a general model that provides new insights into financial contagion and a series of failures between organizations connected through financial interdependence networks. The ultimate goal is to understand the robustness of such networks derived from tree topology which is simultaneously featured with financial interdependencies like crossholdings and multisourcing. In other words, organizations' values depend on each other.

This paper models the reliability of each individual institution at each level of the banking system under the circumstance of financial connections. As a repercussion, an algebraic formula denoting reliability with respect to complexity and multisourcing number was attained. On the premise of maintaining constant variables, this paper also studied the influence of complexity and multivariate number on reliability respectively, which drives more accurate predictions across a wider range of realistic scenarios.

## 2. Research Review

In past studies, the network models of fragility focusing on the forces that make aggregate functionality especially sensitive to the economic environment were presented. The methodologies consider the key structural features of networks that determine their fragility, emphasizing the importance of phase transitions. Then they turn to endogenous decisions and discuss some distinctive implications of fragility phenomena for the equilibria of such models.

There are intricate relationships among different banking institutions. Systemic financial risks can be contagious among banks. The contagion is relatively obvious through financial markets and tail risk channels, and relatively inconspicuous through investor sentiment channels. In general, large state-owned banks are at the core of the risk contagion network, and the risk contagion of some city commercial banks and rural commercial banks cannot be ignored. Large state-owned banks have the highest degree of systemic importance, followed by joint-stock commercial banks. Some city commercial banks and rural commercial banks also have certain systemic importance. The evolution trend of systemic financial risk in China's banking system is highly consistent with the internal and external economic operation and the operation of the banking system.

## 3. Background

### 3.1. Current Chinese Banking System

China's banking system consists of a financial institution, regulatory agencies, self-regulatory organizations and banking cash institutions. The People's Bank of China, beneath the leadership of The State Council, is responsible for formulating and implementing monetary policies, preventing and deactivation cash risks and maintaining cash stability. The China Banking and Insurance regulatory Commission, determined as CBRC, is responsible for the direction of banking cash institutions and their business activities throughout the country.

The China Banking Association could also be a national non-profit social organization registered with the Ministry of civil affairs and a self-regulative organization of China's industry.

China's banking cash institutions embody policy banks (China Development Bank, Export Import Bank of China, Agricultural Development Bank of China), large industrial banks (industrial and full service bank of China, Bank of China, Agricultural Bank of China, China Construction Bank, Bank of Communications), very little and medium-sized industrial banks, rural cash institutions, communication savings bank of China and foreign-funded banks.

The CBRC supervises non bank cash institutions, along with cash and management corporations, trust corporations, enterprise cluster finance corporations, cash leasing corporations, car finance corporations and money brokerage corporations.

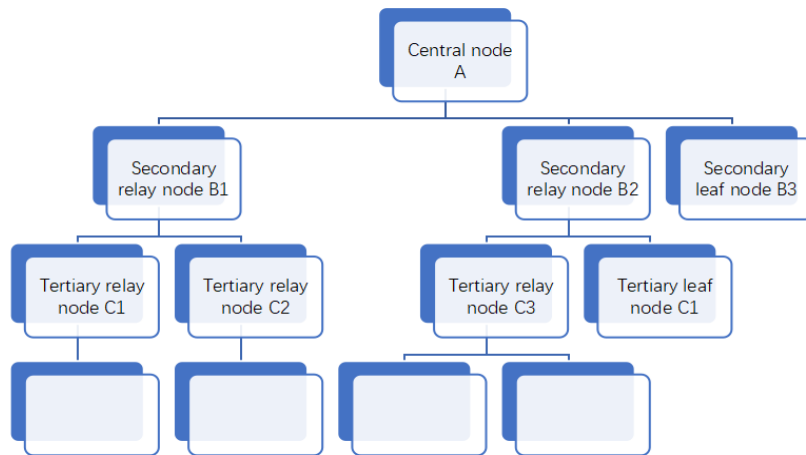


Figure 1: General Structure of Tree Topology.

### 3.2. Tree Topology

**Introduction.** The tree topology evolves from the bus topology and looks like an inverted tree where the top is the tree root. There are branches below the tree root. Each branch can also have branches extending outwards. It is an extension of the bus structure. It is formed by adding branches to the bus network. Its transmission medium will have multiple branches, yet doesn't form a control system. The tree network could be a stratified network. Its structure is often symmetrical, the affiliation is fastened, and includes a bound fault tolerance. Generally, the failure of 1 branch and node doesn't impact the residual branch nodes. the knowledge sent by any node are often unfold throughout the transmission medium, it is also a broadcast network.

There are two kinds of nodes in the system. One is called “the relay node”, which is involved in the financial transactions; the other one is called “the leaf node”, referring to the nodes with outdegrees of 0 and do not expand further.

### 3.3. Methodology

Since the Chinese banking system and tree topology share a high similarity, this paper chooses to apply the latter one to the former one to study its asset and reliability.

Assumptions:

Here this paper studies a hierarchical structure of tree topology with four tiers under the condition of crossholding.

In each bank, 30% of its asset will be held in cross-holdings.

The span of control is two, amounting to the number of subsequent nodes or the banks at the next tier;

- There is one leaf node at each tier;
- The amount of cross holdings between the banks at adjacent tiers are the same;
- Multisourcing;
- Each firm is hit by an independent shock with a probability of  $1-x$  and then fails.

#### 4. Reliability

The four levels from bottom to top correspond to the final financial product  $A_i$ , intermediate product  $B_i$  required to produce A, intermediate product  $C_i$  required to produce B, initial product  $D_i$  required to produce C.

The asset analysis is similar to the asset analysis. There is a certain relationship between the crossholdings of specific firms. Not all potential suppliers can supply a given firm. There is a network among the firms representing who can supply whom.

The shocks are now added to the model. Suppose that firms may be hit by shocks that prevent them working functionally. The probability of each firm undergoing a shock is  $1-x$ , with a probability of  $x$  it is unshocked. The variable  $x$  is called the firm's strength; it measures how resistant a firm is to idiosyncratic shocks. For each firm to be able to produce, it must be unshocked and be able to source all its required inputs. If a firm is able to produce, we call it functional. The firm's reliability is defined as the probability that it is functional. In our first example, firm  $a_1$  is functional with a probability equal to  $x^3$ .

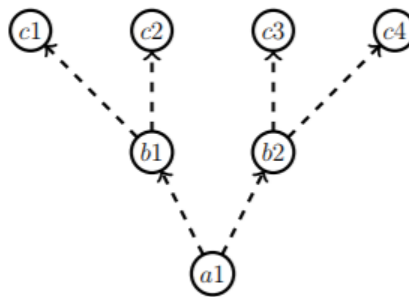


Figure 2: Simple Financial Networks.

#### 4.1. Algebraic Model

To explain the qualitative transition, this part presents a simple calculus for computing reliabilities at any level. Consider the event that a firm at an arbitrary level  $l$  can produce. In the trivial case where the firm is at level 0 and has no specific supply relationships (i.e., only needs to source standard off-the-shelf inputs), we will stipulate that this is  $\rho(x, 0) = x$ : the firm operates only if it is not exogenously shocked. For a firm at level  $l \geq 1$ , it is functional if it is unshocked (probability  $x$ ) and the tier it belongs to is functional, which means no more than one node at this tier is shocked. The upstream tier (at level  $l - 1$ ) should also be functional.[3]

Each firm is hit by an independent shock with probability  $1-x$ . To find  $a_1$ 's reliability [7]:

The probability that neither  $c_1$  nor  $c_2$  is functional is:  $(1 - x)^2$

The probability that either  $c_1$  or  $c_2$  is functional is:

$$1 - (1 - x)^2$$

$B_1$  is functional if:  $c_1$  or  $c_2$  is functional and it is unshocked.

So with probability:  $y = x[1 - (1 - x)^2]$

The probability that neither  $b_1$  nor  $b_2$  is functional is:  $(1 - y)^2$

The probability that neither  $b_1$  or  $b_2$  is functional is:

$$1 - (1 - y)^2$$

The probability that a1 is functional is:

$$x[1 - (1 - y)^2] = x(1 - (1 - x(1 - (1 - x)^2))^2)$$

### 4.2. Formula Derivation

Since each of these is functional independently with probability  $\rho(x, l-1)$ , this gives rise to the equation[6]:

$$\rho(x, l) = x[1 - (1 - \rho(x, l-1))^n]$$

We will inspect this type of equation and its mechanics in a more general version of the model in the next subsection, but for now we can derive one intuitive consequence. It is apparent that  $\rho(x, l)$  is decreasing in  $l$ , so it converges as  $l \rightarrow \infty$  to some number  $r = \rho(x)$ , which we can think of as “ $\rho(x, \infty)$ ,” which is the reliability of both a firm and its suppliers when  $l$  is large. That number satisfies

$$r = x[1 - (1 - r)^n]$$

### 4.3. A More Practical Consideration

Suppose each of the firms is functional with a probability  $r$  independently.

Let  $R_x(r)$  be the probability that you are functional,

$m$ —complexity, the number of inputs needed for each product

$n$ —multisourcing number, number of potential suppliers of each needed input

The probability of the given supplier is available  $R_x(r)=xr$

The probability of the given supplier is not available  $R_x(r)=1-xr$

The probability of all suppliers of a given input not available  $R_x(r)=(1 - xr)^n$

The probability of there is a supplier of a given input available

$$R_x(r)=1 - (1 - xr)^n$$

The probability of there is a supplier of all inputs available [2]

$$R_x(r)=[1 - (1 - xr)^n]^m$$

### 4.4. Crossholdings into Consideration

Each tier is regarded as a continuum. It is functional if no more than one node at this tier is unfunctional. If more than one node at this tier is unfunctional, the whole tier is unfunctional.

As a criterion, one node can only work if the previous tier is functional due to cross holdings.

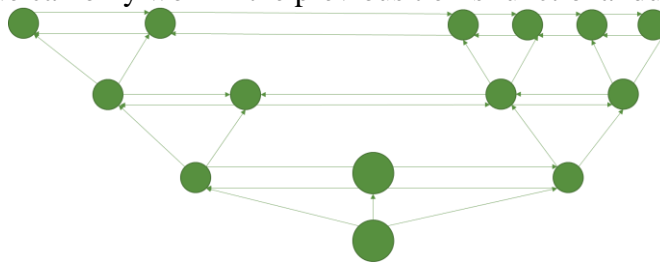


Figure 3: Financial networks in crossholdings.

### 4.5. Derivation+

$$P(\text{Di functional})=x$$

$$P(\text{TierD functional})=x^6+C_6^1 (1-x) x^5$$

$$P(\text{Ci functional})=[1 - \left[ (1-x) \right]^2] \left[ x \right]^6+C_6^1 (1-x)x^5]=y$$

$$P(\text{TierC functional})=(1-x)y^4+C_3^1 x(1-y) y^3+xy^3=z$$

$$P(\text{Bi functional})=$$

$$[1 - ((1-y))^2] [((1-x)y)^4 + C_3^1 x(1-y)y^3 + xy^3]$$

$$P(\text{Tier B functional}) = (1-x)z^2 + xz^2 + C_2^1 xz(1-z)$$

Therefore, we can get the probability of node Ni is functional:

$$P = [1 - ((1-xr))^n]^m [x^{2+2^{(n-2)}} + C_{(2+2^{(n-2)})}^1 (1-x)x^{(1+2^{(n-2)})} + xr^{(1+2^{(n-2)})}]^m$$

r: The suppliers are functional with probability r independently

m=complexity      n=multisourcing number

#### 4.6. Study of Relationship of n-P (multisourcing number and reliability)

When m=4, x=0.9, n=1/2/3/4/5:

Use Matlab to plot the corresponding r-P lines and the equilibrium line  $R_x(r) = 1 - (1 - xr)^n$

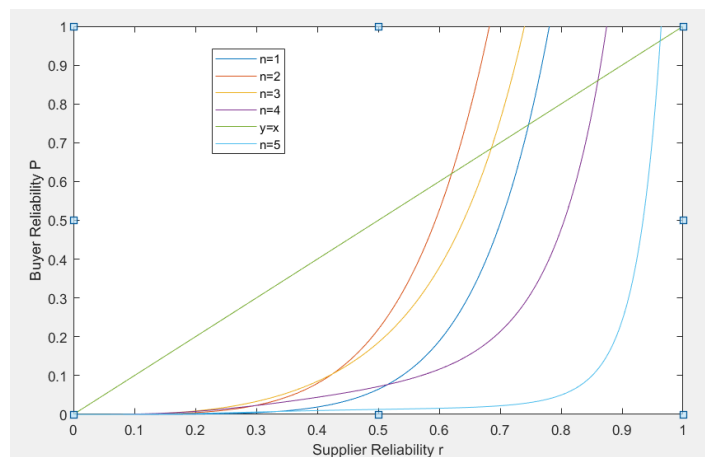


Figure 4: The relationship of n-P.

#### Results Analysis

With the augment of multisourcing number, the reliability also rises.

The curves concave up, which denote that the speed of transformation increases.

For any fixed value of x, we simply look at the largest crossing the curve on the right-hand side with the 45-degree line, as show in figuer 5. Note that when x is sufficiently low, the only intersection is at 0, then, as x increases, a higher intersection (near zero) emerges.

The kink in the probability of successful production around the threshold of 0.5 is a continuous or first-order phase transition++: at a certain critical point, the dependence of  $\rho$  on x qualitatively changes. From a planner's perspective, increasing the strength of all nodes has zero marginal value for  $x < 0.5$ , and suddenly quite a large marginal value for x just above 0.5 [4].

The relationship between n and P, though not proportional,

When  $n \leq 3$ , the reliability is maximum at  $n = 1$

When  $n > 3$ , the bigger the n is, the greater the reliability is.

#### 4.7. Study of Relationship of m-P (Complexity and reliability)

When m=1/2/3/4/5, n=1, x=0.9

Use Matlab to plot the corresponding m-P lines and the equilibrium line  $R_x(r) = 1 - (1 - xr)^n$

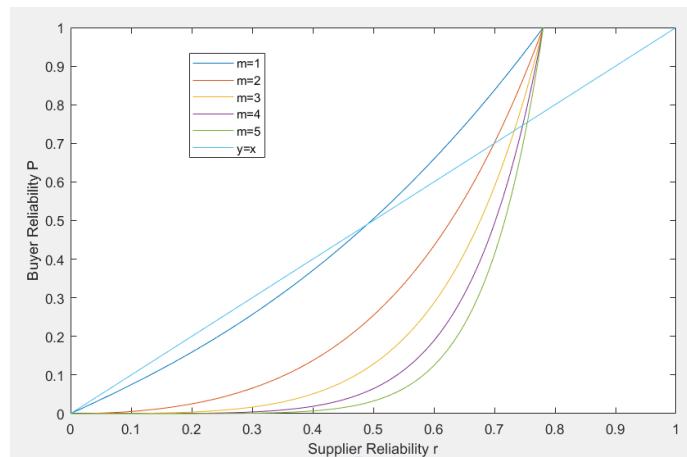


Figure 5: The relationship of m-P.

### Results Analysis

With the augment of multisourcing number, the reliability also rises.

The curves concave up, which denote that the speed of transformation increases.

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The kink in the probability of successful production around the threshold of 0.5 is a continuous or first-order phase transition: at a certain critical point, the dependence of  $\rho$  on  $x$  qualitatively changes. From a planner's perspective, increasing the strength of all nodes has zero marginal value for  $x < 0.5$ , and suddenly quite a large marginal value for  $x$  just above 0.5. The intersection of the equilibrium curve and the curve of  $m=1$  intersects at  $x=0.5$ .

This is related to the emergence of a giant component in an Erdős-Rényi random graph—indeed, this is really a version of the same phenomenon. The transition has obvious economic implications. [8]

The bigger the multisourcing number is, the greater the buyer reliability is.

## 5. Conclusions

This paper develops a model depicting the reliability of individual banks in the financial networks and at the same time under the circumstances of crossholdings and multisourcing, combining tree topology with the Chinese banking system.

At last, this paper have drawn many analogies between supply networks and financial networks. However, there is one notable difference regarding our knowledge of their structures. On one hand, the model implemented is still a quite ideal and simple one. In real world, the shocks may result in regional chaos and disruptions. On the other hand, the multisourcing relationship between the banks is far more sophisticated than the model.

In the case of financial networks, there is a large body of work planning the structure of the network's various financial markets. These measures are the cornerstone of many of the financial network papers this paper have cited, and they have not only guided the theoretical work but have also energized the literature. A similar knowledge base is largely missing in supply networks. This paper does not imply much about the overall structure of supply networks. Work that provides a more rigorous understanding of the empirical structures of supply networks, how these structures differ across industries, and how these structures are subject to shocks and change would be very welcome.



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