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Research on the Development of Implied Volatility in Option Pricing

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Abstract: Option, as an important financial derivative, has received more and more attention from investors in recent years. This paper includes three parts, the first part introduces the concept of options, including the birth and history of options, the trading of options, and different types of options. Then it discusses the history of option pricing and two typical and classic option pricing models and then introduces implied volatility. The second part of this paper introduces the specific development of the option pricing models, the correction process of option pricing models, and volatility models. Different types of volatility, and their comparisons are included. And this paper also discusses the two functions of implied volatility, one is predicting the future and the other is for risk management and portfolio management. The third part concludes the first two-part and discusses the recent progress of option pricing and its volatility.

Keywords: option pricing, Black-Scholes model, implied volatility

1. Introduction

1.1. History of Option

An option is a kind of contract, which involves buyers and sellers. And the option holder has the right, but not the obligation, to buy or sell the option. According to historical records, the earliest record of option trading happened in ancient Greece. Thales of Miletus predicted the olive will have a good harvest in the coming spring. Miletus negotiated a price with the farmer for the right to use the olive oil press during the next harvest. The next year, the olive had a great harvest, Miletus then rented the right at a higher price to other farmers to make a profit [1]. Later in the 19th century, the Over the Counter (OTC) option was raised. The Chicago Board Option Exchange (CBOE) began trading stock options in 1973 [2]. Compared to the OTC market, CBOE formulated to use standardized option contracts and developed a trading system. And CBOE set up an intermediary agent company to deal with the clearing and delivery of option trading in a centralized manner, to ensure the option contracts can only be traded on guaranteed exchanges, thus providing a reliable guarantee for the trading and execution process of options. The appearance of CBOE vastly promoted the development of the option market.

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1.2. Different Types of Options

According to different criteria, options can be classified in different ways. Options can be categorized as call options or put options based on the various rights and obligations of the buyers and sellers. Depending on whether the options can be exercised before maturity, European options and American options are the two primary categories. Only on the expiration date may European options be executed, while American options, allow for exercise on any trading day prior to the expiration date. According to trading places, options have Floor Traded Options and Over the Counter Options. According to the subject matter, financial options can be divided into exchange rate options, stock options, and interest options.

1.3. History and Introduction of Option Pricing

Since option products were introduced, the issue of option pricing has been the core of options trading. Option pricing theory values option prices in the market theoretically by using variables such as stock price, strike price, volatility, interest rate, and maturity. Option pricing theory was first proposed in 1900 by Louis Bachelier, the father of modern option pricing theory. He derived a formula for European call option in his Ph.D. thesis, he assumed the stock price follows a normal distribution, and this made the probability of negative stock prices greater than zero, which is impossible [3]. Also, Bachelier's model ignores the conditions of the real stock market. Later in 1961, Sprenkle started to assume lognormal returns based on Bachelier's formula, but in Sprenkle's assumption, the degree of risk aversion and the stock price's drift rate need to be reckoned, which was quite tough [4]. Boness improved the formula [5]. The formula he derived is to discount the stock price at expiration to calculate the time value of money. After that, until 1973, many scholars improved and proposed new option pricing models. But, since some parameters in these models are subjective, none of these models had practical reference significance and were not universally accepted. In the 1970s, the proposition of the Black-Scholes Model started the revolution of the option pricing field. In 1973, Black and Scholes published *The Pricing of Options and Corporate Liabilities* and won the Nobel Economics Prize. According to the B-S option pricing model, stock prices move in a way that is geometrically Brownian and has constant volatility [6]. It also follows that：

No dividends or other profits are Markets are paid.

The option is the European option.

There are no fees associated with buying or selling stocks or options.

Prior to maturity, the risk-free rate remains constant.

There is no risk-free arbitrage opportunity.

The return follows a normal distribution.

Black-Scholes Model led a to prosperity because more and more investors were willing to invest in options. But the B-S model still has limitations. Additionally, Merton provided some thorough mathematical explanations of the Black and Scholes technique and expanded the model by taking dividend payments and strike price fluctuations into account [7]. Their extraordinary work had a huge impact on options pricing, which helped options re-establish the stellar reputation that made trading options so popular today.

Besides the most famous B-S model, another meaningful option pricing model is the Binomial Pricing Model (BOPM). BOPM was proposed by William [8]. In this model, the time to expiration is broken down into several intervals. At each small period time of interval, the stock will either go up or go down and BOPM has the following assumptions:

Transaction costs and taxes are 0.

The risk-free rate remains constant.

Shares have no dividends.

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volatility remains the same.

Borrow at a risk-free rate.

This model uses the backward tracking method to determine the original option price in reverse.

1.4. Introduction of Implied Volatility

In the option pricing model, the only unknown variable is volatility. And in the actual financial market, volatility is deterministic and cannot be observed directly. In the B-S model, the option price was determined by five variables, the current stock price, the time to expiration, the risk-free rate, the strike price of an option, and the volatility. If the option price is known, The B-S model can be inverted to determine implied volatility. In actual option trading, implied volatility is used to predict option prices and future markets. Volatility, therefore, has a major impact on the option market. In practice, there are some specific characteristics when discussing relationship between volatility, stock price, and option price:

- If all the other conditions are equal, an option with shorter maturity will have higher implied volatility [9].
- If the stock price is relatively high, the volatility, on the contrary, will be relatively low, which makes the possibility of stock upward fluctuation smaller; conversely, when the stock price is low, volatility increases and the stock is more likely to fall. Compared to the log-normal distribution, the probability distribution of real actual stock price has a higher left tail, and lower right tail.

2. Development and Literature Review

Volatility is traditionally calculated by using the standard deviation of the logarithmic returns of a security over time. For the calculation types of volatility, there are various intricate types. One is using historical information to predict future volatility, as the historical information method, which is also the time series analysis of volatility. This method includes the ARCH family of models, and the Stochastics Volatility models. The other type is based on the strike price of the option to deduce the market's expectation of future volatility, this volatility calculated is the implied volatility. The comparisons of these two types of volatility have always been concerned and discussed.

Since the introduction of the Black-Sholes model, the issue of option volatility has always attracted a lot of attention and discussion. In the classic Black-Scholes model, the volatility is assumed to be a constant number. However, empirical research shows that the volatility of an option is following a stochastic process and varies over time [10]. The strike price, time to maturity, and the current price of a stock can be observed directly from the market, the volatility is the uncertainty factor in the Black-Scholes model. The discrepancy between the actual volatility and the volatility in the B-S model has led researchers constantly revise the volatility formula based on the development and phenomenon of the financial market. Also, the B-S model cannot explain the "volatility smile", which led to an error in the option pricing [11]. The phenomenon of volatility smile skew made many researchers abandon the assumption of fixed volatility in the B-S model and they started to assume volatility as a random variable that obeys certain random processes.

In 1976, Merton produced the first option pricing models with changing variance [12]. He abandoned the hypothesis that returns obey normal distribution in the B-S model, and introduced the Poisson process, he put forward the jump-diffusion model and set jump risk as a non-system risk that obeys normal distribution. The introduction of the Poisson process improved the practicability of the classic BS model. In the same year, Black found the leverage effect. Black's leverage effect hypothesis postulated a distribution between stock returns and volatility, which is negatively correlated [13]. Later Cox & Ross & Rubinstein proposed the Binomial Option Pricing Model under a discrete time [14]. This model is simple to understand and can be used to price complicated options.

Engle first used the autoregressive conditional heteroskedasticity (ARCH) model to characterize the autocorrelation and persistence of volatility [15]. In this non-constant volatility model, the aggregation effect of volatility is considered, and Engle won his Nobel Price. Boleslaw proposed generalized autoregressive conditional heteroscedasticity (GARCH) to improve the ARCH model, he thinks the price changes depend on both past price and past volatility [16]. Nelson considered the leverage effect of volatility and proposed an exponential generalized autoregressive conditional heteroskedasticity model (EGARCH) [17]. There are also improvements to GARCH models such as the GARCH-GH model and the GARCH-Gaussian model. These time series analysis models show that volatility responds with significant asymmetry, in the face of unexpected stock price declines, volatility is usually adjusted more than the incline of stock price.

The CIR model was proposed by Cox, Ingersoll, and Ross [18]. CIR model has the property of mean reverting, and it says that the interest rates move around an average, if they deviate from the average, they will always return to the average value. This model avoids negative interest rates. Hull and White first proposed the Stochastic Volatility (SV) model [19]. They used the Taylor expansion and derived an option pricing formula for the call option. They assumed that the stochastic volatility of stock has zero correlation with the stock price for a European call option, then the price calculated from the B-S model will be inaccurate, the price of the out-of-the-money or in-the-money option will be undervalued, which has great significance in the option pricing theory. Hull and White assumed of CIR process but did not take the relationship between expectation and variance into consideration. Stein and Stein created another Stochastics Volatility model, which used a mean-reverting process, assuming a zero correlation between returns and volatility [20]. However, Stein and Stein's model is insufficient for instantaneous volatility, and it could not avoid negative volatility.

The Heston model, put forth by Heston, is a typical and well-known stochastic volatility model [21]. The Heston model is a closed-form solution for European option pricing, and it has five parameters. It considers of leverage effect with no negative variance, and it incorporates the leverage effect. Heston's model uses the CIR process to characterize the expectation and variance respectively and assumes that the Brownian motion corresponding to the expected variance has zero correlation. Heston pointed out that volatility satisfies non-negativity and mean-reversibility and used of affine structure model to calculate a closed solution for the European option, to prevent system errors. If considering the impact of the five parameters in the Heston model on implied volatility: the mean-reverting speed is negatively correlated with the curvature of the volatility "smile", while the volatility variance is positively correlated with the curvature of the "smile". The increase of the third and fourth parameters, the long-term volatility, and the initial variance will shift the implied volatility "smile" curve moves upward, and the correlation coefficient of stock price and volatility stochastic process is related to the slope of the "smile" curve [22]. The Heston model is one of the most important models in the option pricing models in the continuous-time stochastic volatility models. It explains the "smile" and "skewness" phenomena of implied volatility. Also, the closed-form solution for the European option made the pricing process more analytical processable, and it has become a popular model. Bates combined the good aspects of the Heston model and Merton's jump model and raised jump diffusion and stochastic volatility model (SVJ).

Another important model called Local Volatility was proposed by Dupire. This Local Volatility was achieved by putting volatility as a function of time and forward price. The stochastics of volatility is entirely deduced by the asset prices. The volatility under the local volatility is not only stochastic but also retains the completeness of the market under the B-S model. Kani et al. considered local volatility to be an estimate of a random uncertainty in every instantaneous future volatility [23]. He constructed a portfolio related to index options in underlying volatility, to hedge against future changes in local volatility, reducing or to some extent, increasing volatility risks. Later Sepp.

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gave the analytical formula of local volatility based on implied volatility according to the trading habits of the financial market [24]. the Local Volatility has only one random variable, it is simpler to calculate compared to the Stochastics Volatility models. It also has the advantage that it accurately fits the market volatility data. It should be considered that the Local Volatility model is not an isolated model, Gyongy built a bridge between the two kinds of models, while also adding a local-volatility-related approach to the computation of stochastic volatility [25].

In actual option trading, implied volatility can be used to predict future options and point out the direction of adjustment to the market. Based on previous studies on volatility, Mixton constructed a multi-factor S&P 500 index model and concluded that the factor causing volatility skew was likely to be the government debt spread [26]. Chan & Cheng & Lung considered the behavior of traders and found that when the strike price was and the liquidity speed was high, the demand for the put option increased greatly. And the index option showed an asymmetric effect on the implied volatility. John Hull found that the implied volatility under risk neutrality was smaller than the lognormal distribution under the same condition [27]. Hull also explained that the increase in leverage was caused by the decline in stock prices, which reduced the risk of the company's equity assets. However, the rise of stock prices will increase the risk of the company's assets and reduce the leverage ratio, thus reducing the volatility.

Besides the above application of volatility in option pricing. Another application of implied volatility is an effective tool for risk management and portfolio management. After the appearance of the B-S model, the financial market partitioners hedged the risks in the market by construing an option, which led to the emergence and improvement of the VIX index and the study of option risk evaluation parameters. The volatility index (VIX) was launched by CBOE in 1993, this index was introduced to calculate option prices based on the B-S model and GC model, and the VIX index is a 30-days implied volatility index of S&P 500 option contract prices. In 2003 the VIX was improved. The S&P500 index was used in place of the S&P100 index by the VIX, which also obtained the S&P500's strike price, including the put and call options, to represent more market data. VIX futured by CBOE in 2004, And in 2006, CBOE began trading VIX options, which provided investors with financial products that use volatility to invest [28]. It provides a benchmark for estimating short time volatility. VIX is also known as the investor fear gauge. Whaley found that investors' greed in the market was manifested in underestimating risk and overestimating optimism [29]. The Delta value is a risk evaluation parameter [30]. The Delta parameter refers to the ratio of the change in derivative value to the change in the asset and is the slope of the relationship curve between derivative value and the underlying asset price.

3. Conclusion

In general, the option theory and its volatility calculation are constantly changing with the development of the market. In addition to the mentioned volatility model and option pricing model above, many scholars have proposed to establish relevant models. Due to the complexity of various models, the other specific development will not be included in this paper.

For the prediction of volatility, either using the historical volatility by the time series or using the implied volatility method has its own advantages. Generally speaking, the time series analysis is more theoretical by using the models, while the implied volatility is more about the extraction of real-time market information. Relatively speaking, the short-term volatility is more suitable to adopt the time series model. On the contract, the volatility forecast over one month often adopts the implied volatility method.

In order to obtain more accurate option price trends, nowadays, scholars combine machine learning to study option pricing, including the Monte Carlo simulation and neural network models. Monte Carlo relies on repeated sampling and statistical analysis for the valuation of options, and the ad-

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vantage of Monte Carlo techniques is its flexibility and Monte Carlo method could reduce the variance. Deep neural network models have the strong nonlinear fitting ability and feature capture ability and have attracted more and more attention in the field of time series analysis. But the use of the neural network pricing model alone still has disadvantages. The deficiency is manifested in the lack of interpretation of the NLSTM modelling process. In this regard, NLSTM and classical option pricing theory can be combined to make up for each other's shortcomings and do hybrid modeling. Moreover, the price of options in the market is also affected by other various external factors, such as market panic and the overall economic situation. If these factors are considered, more in-depth research is needed.

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